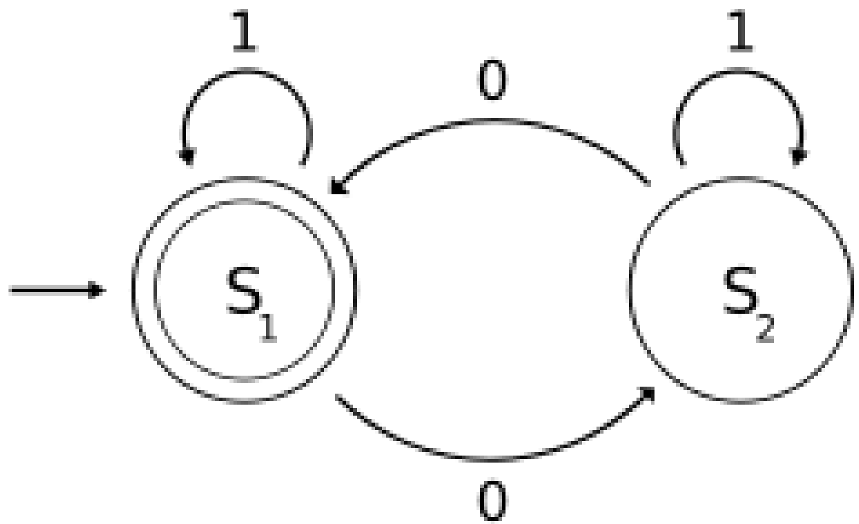
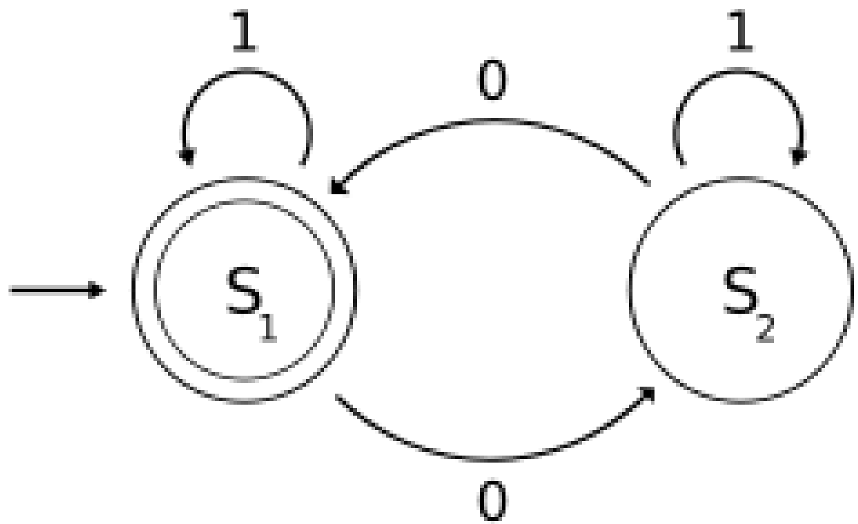
Automata


1
0
1
S1
S2
0



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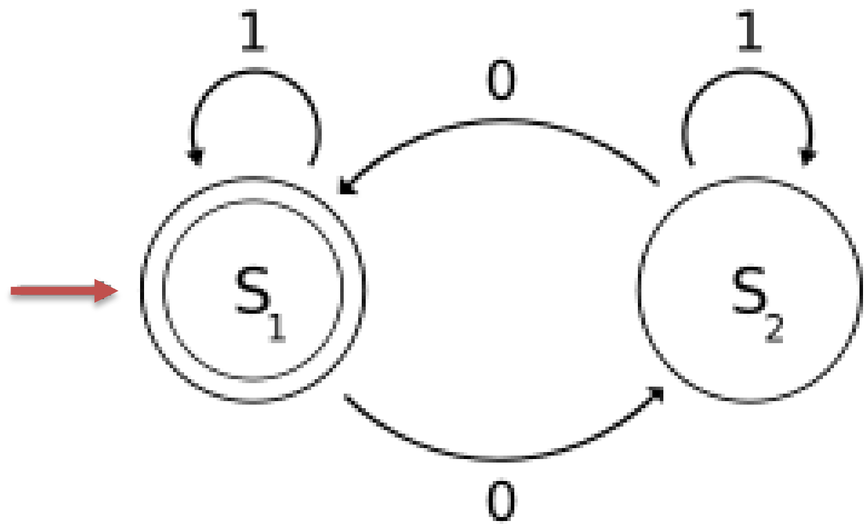

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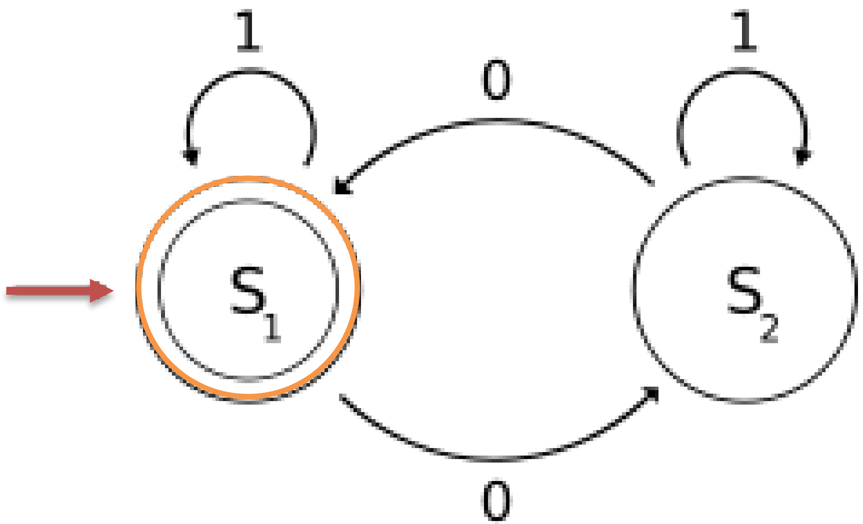

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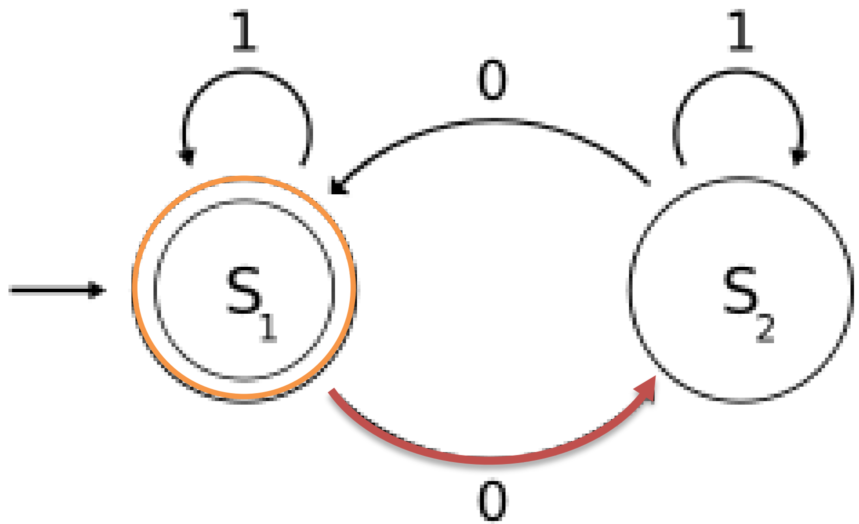

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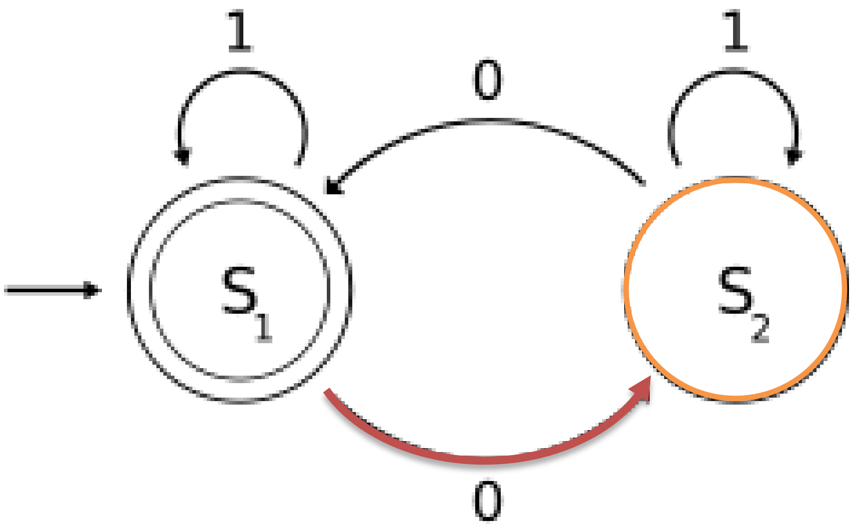

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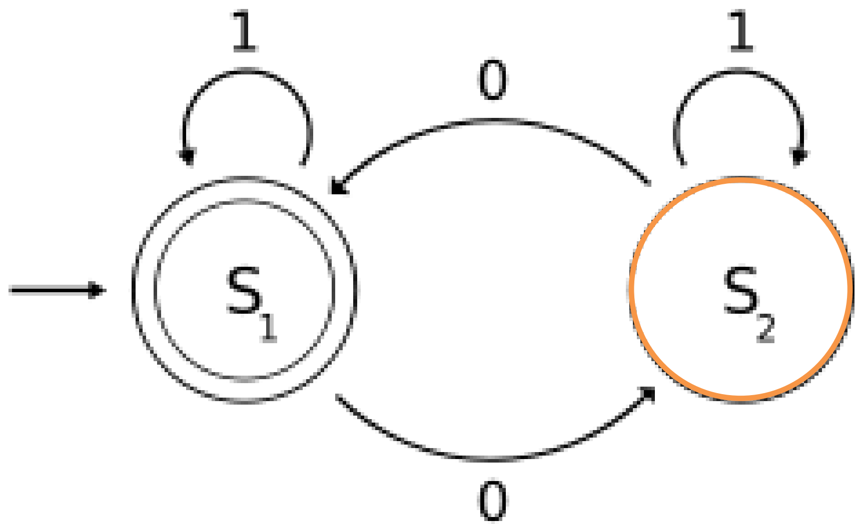
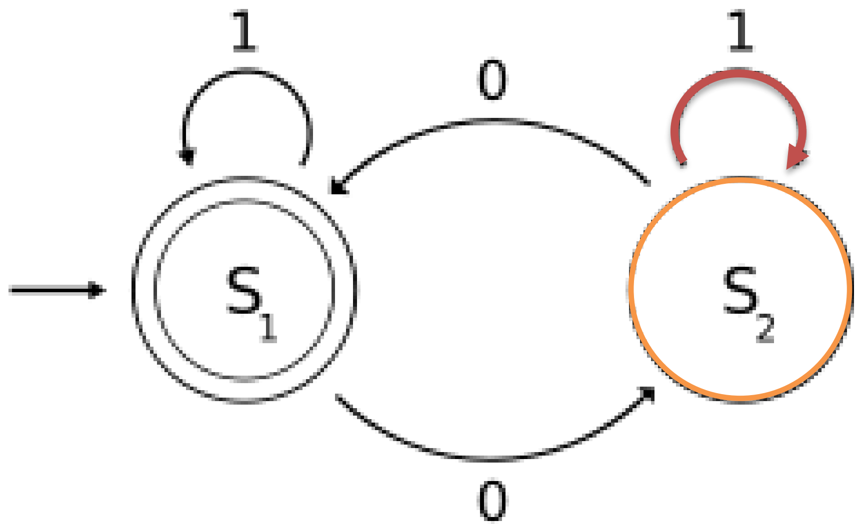


'0101010'



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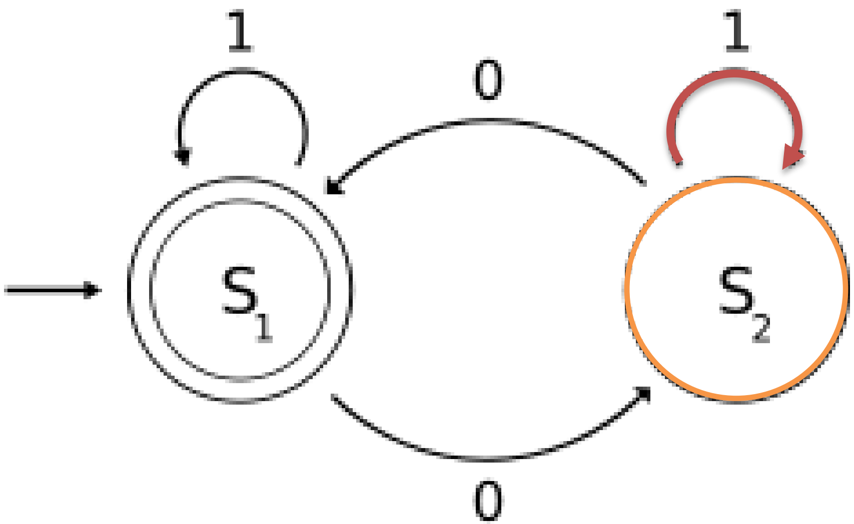
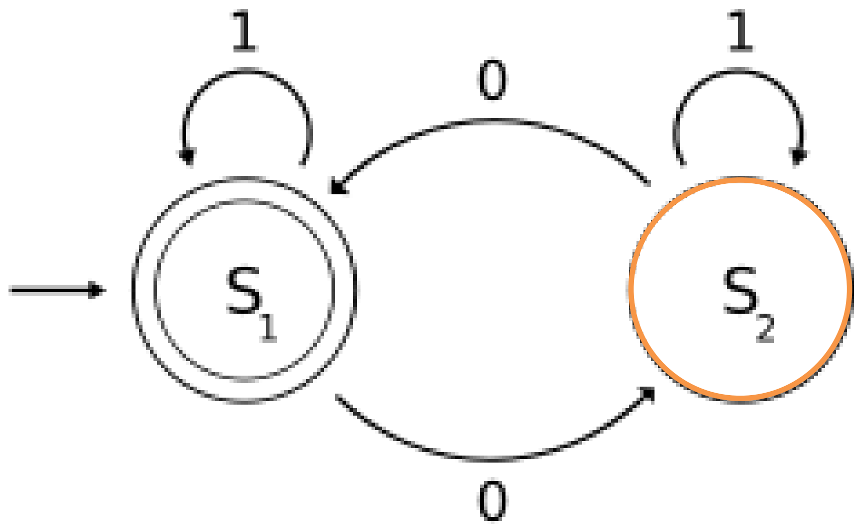
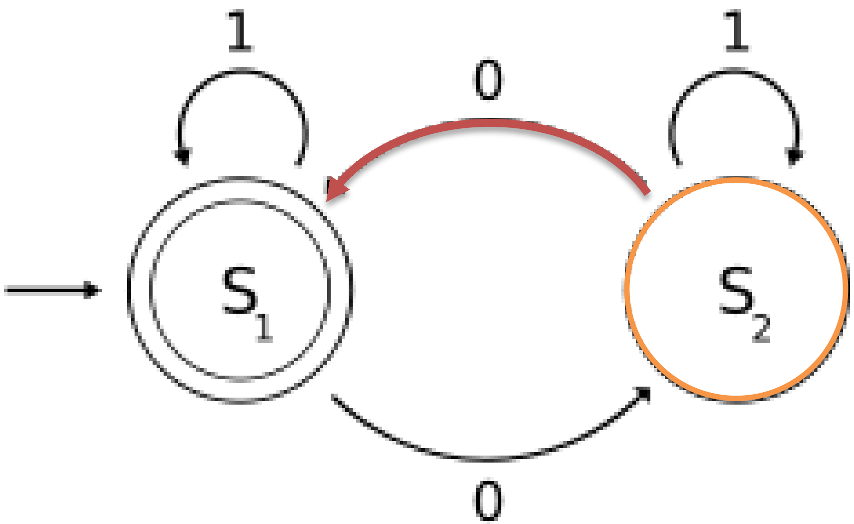


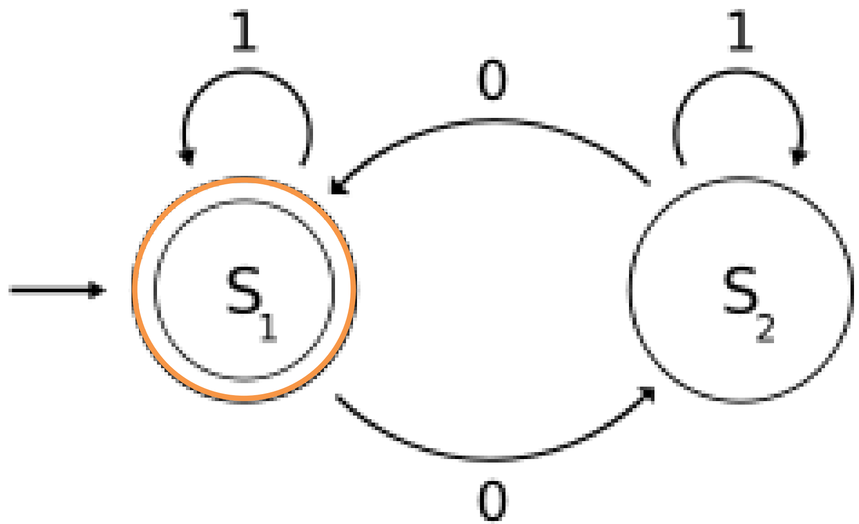
‘0101010’



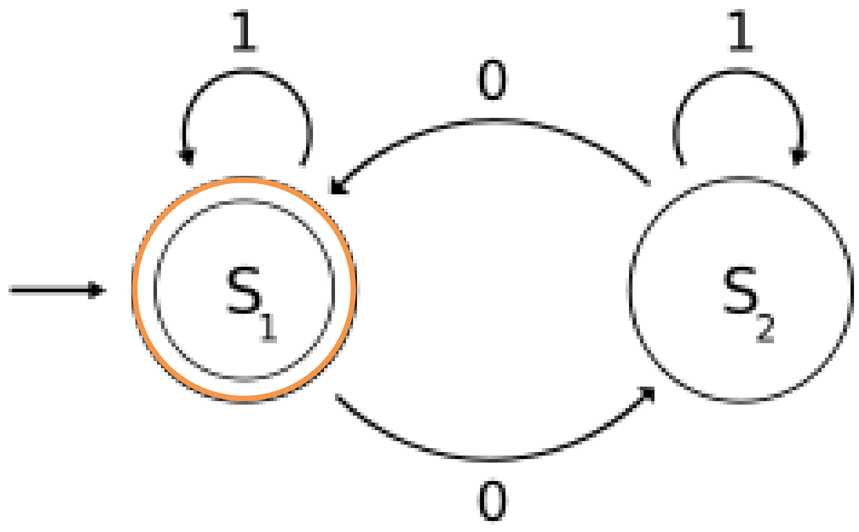
‘0101010’



‘01|01010’



‘010 1010’



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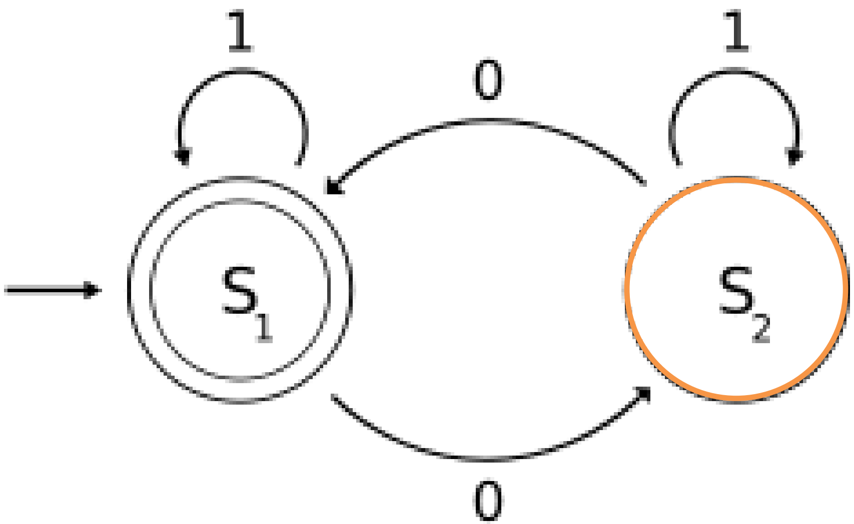
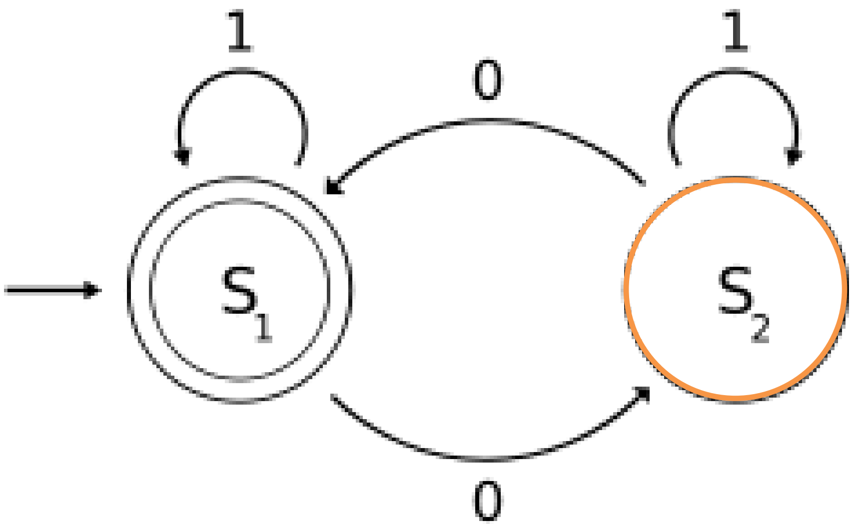


‘0101010’



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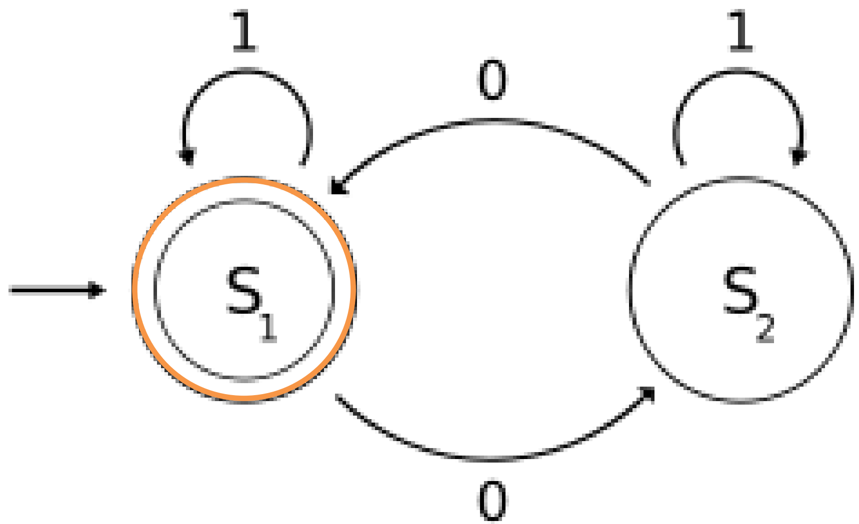
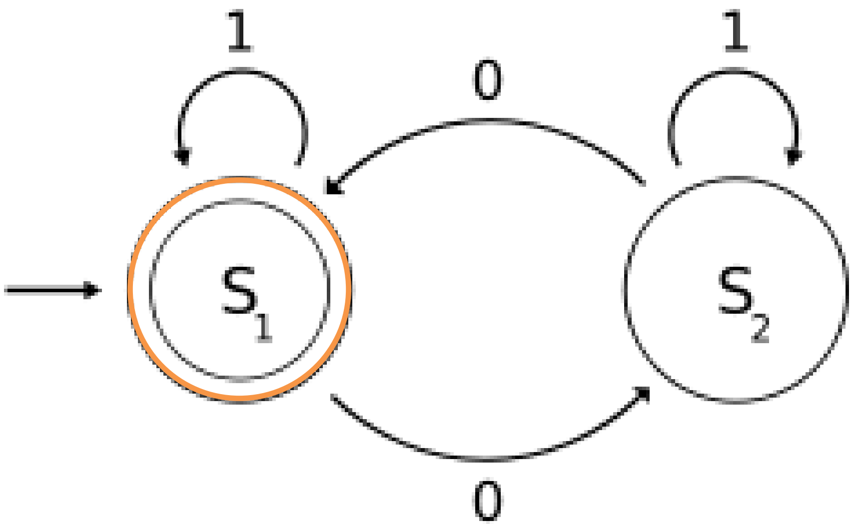


'0101010'



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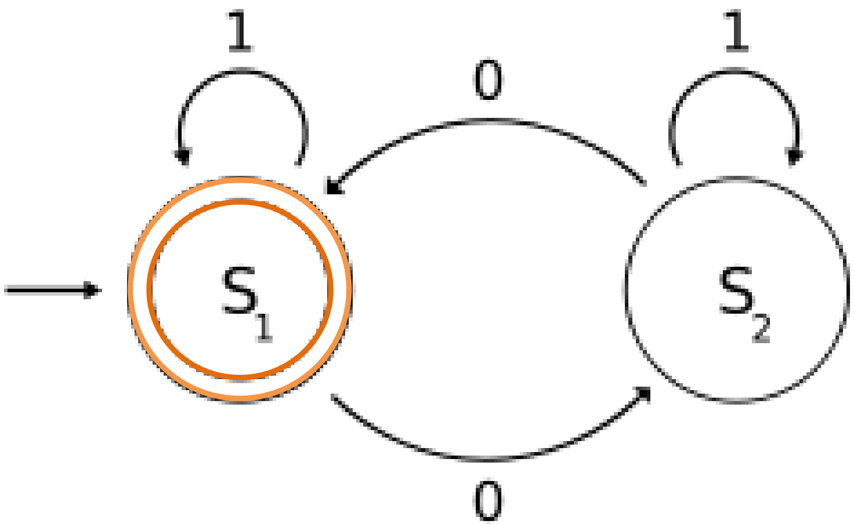

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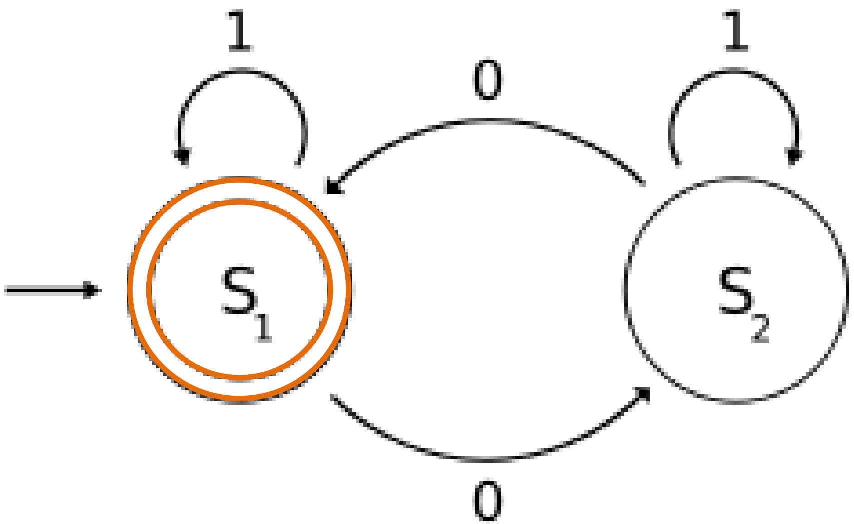
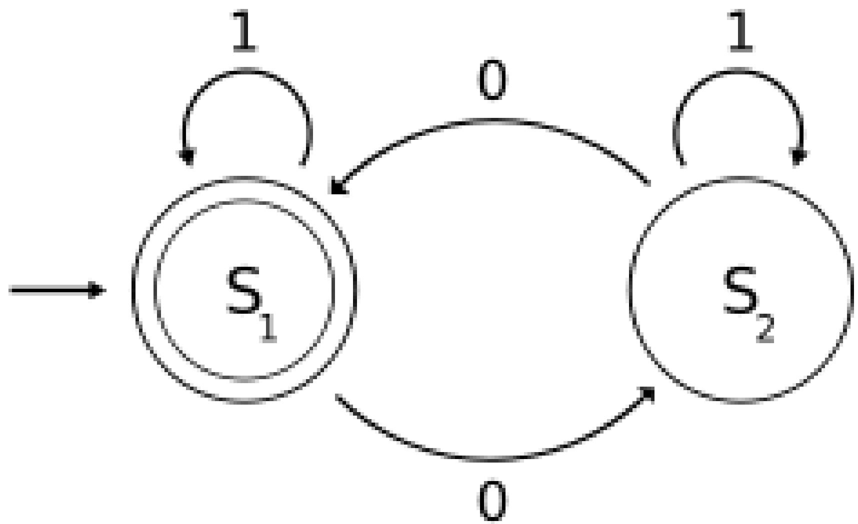
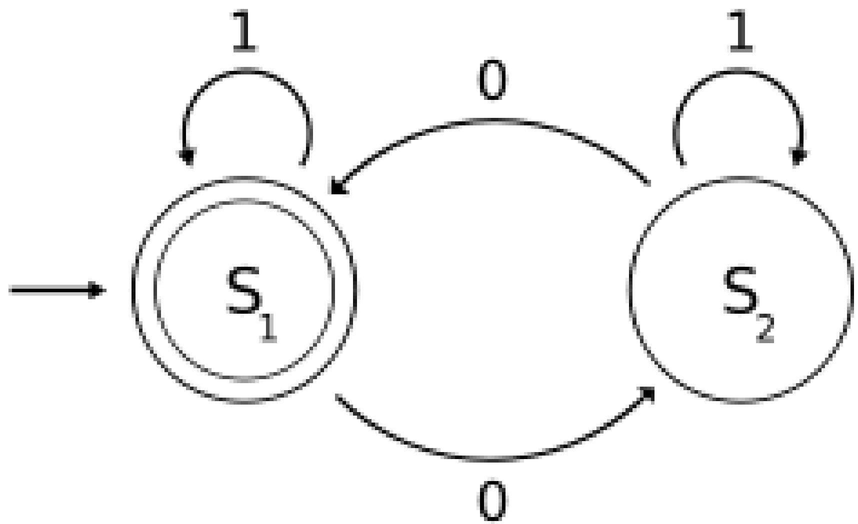


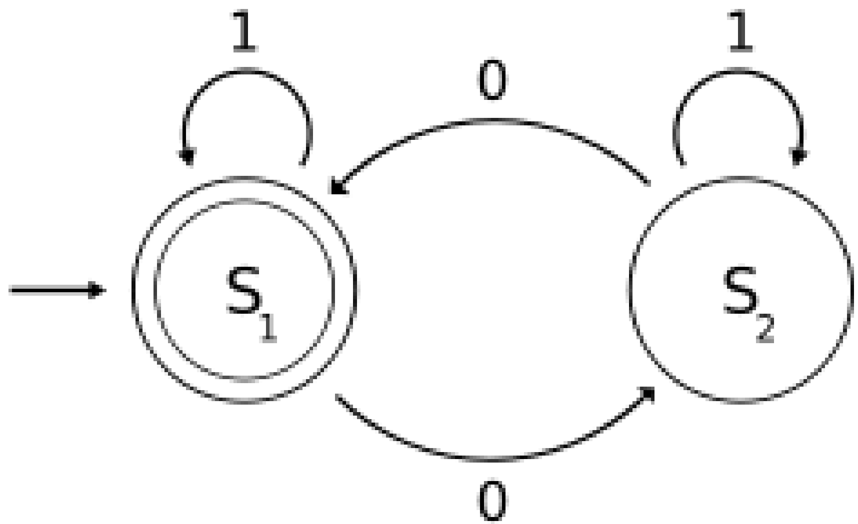
‘0101010’



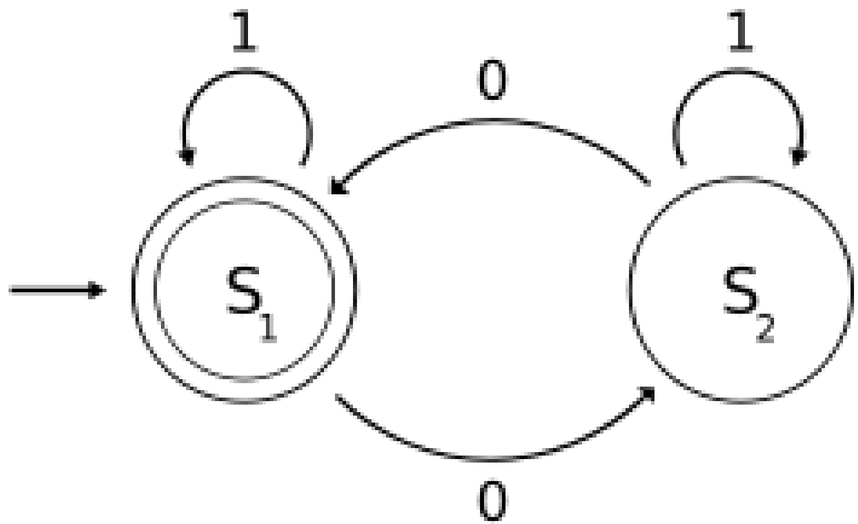
01111?



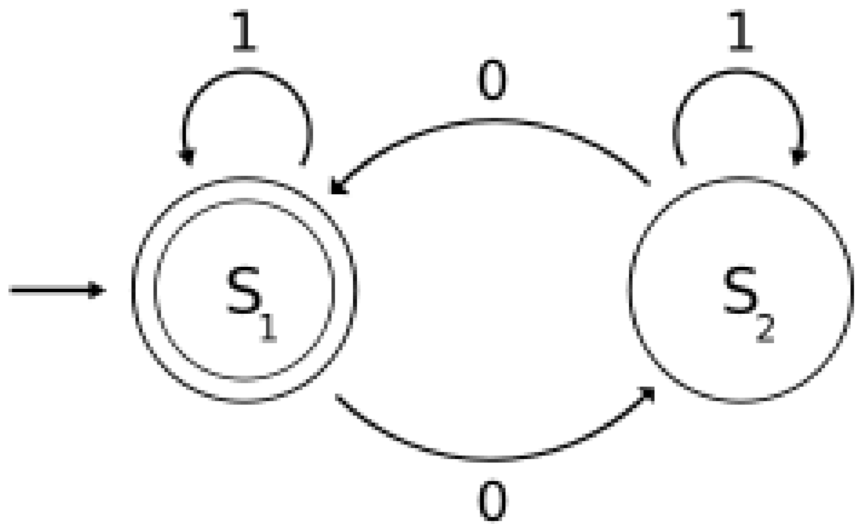
1100?



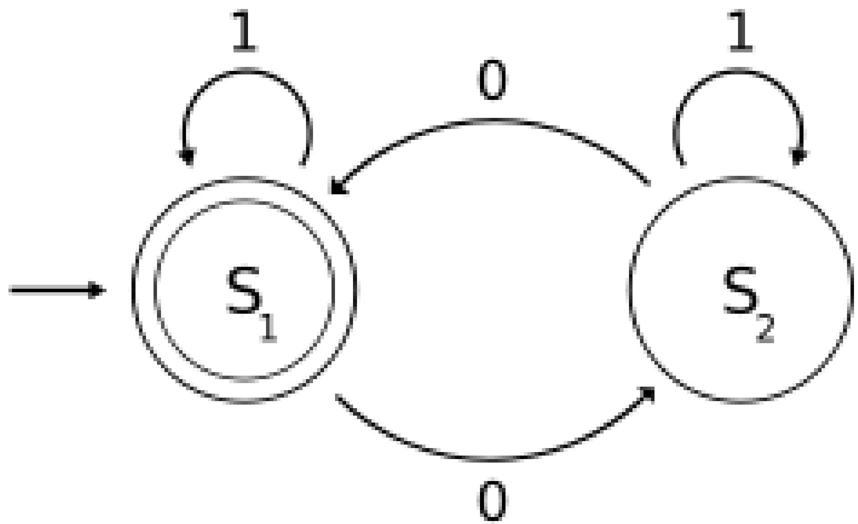
11010?



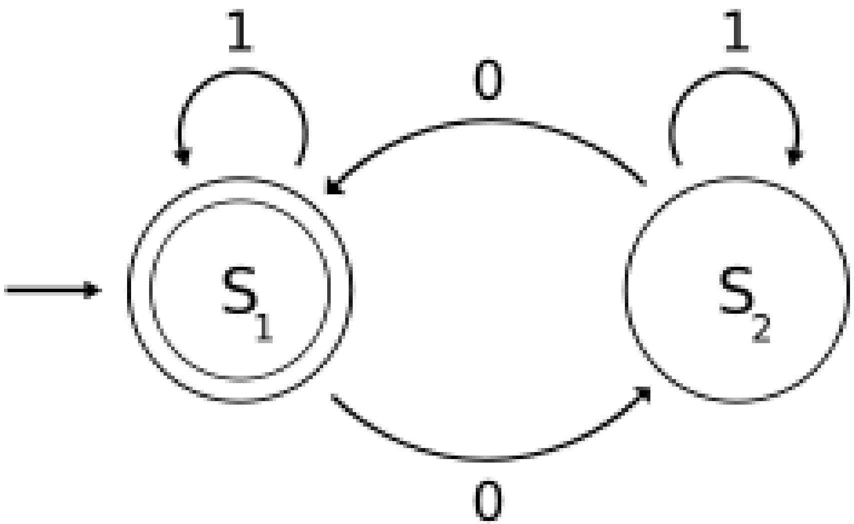
000?



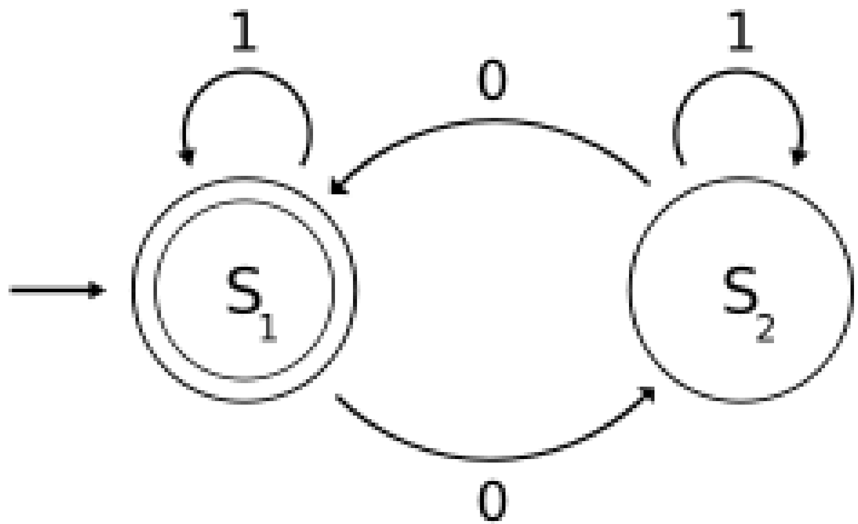
00110?



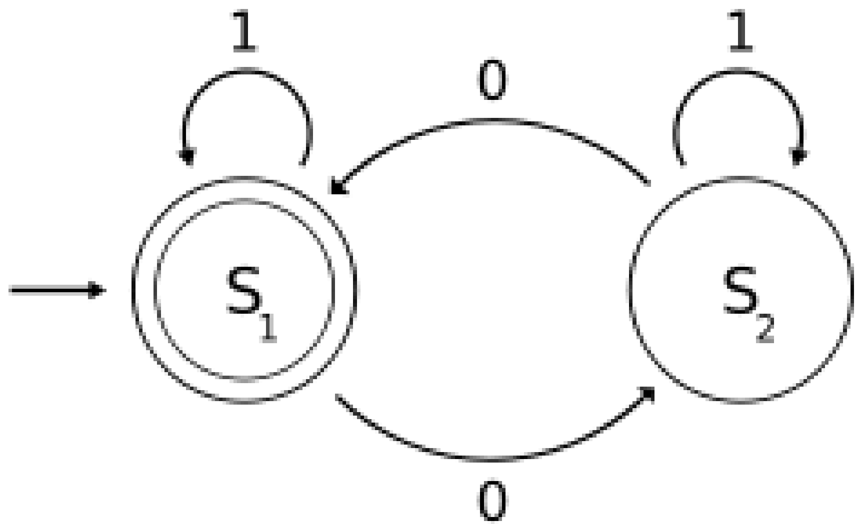
Accept	Not Accept
0101010	01111
1100	000
11010	00110



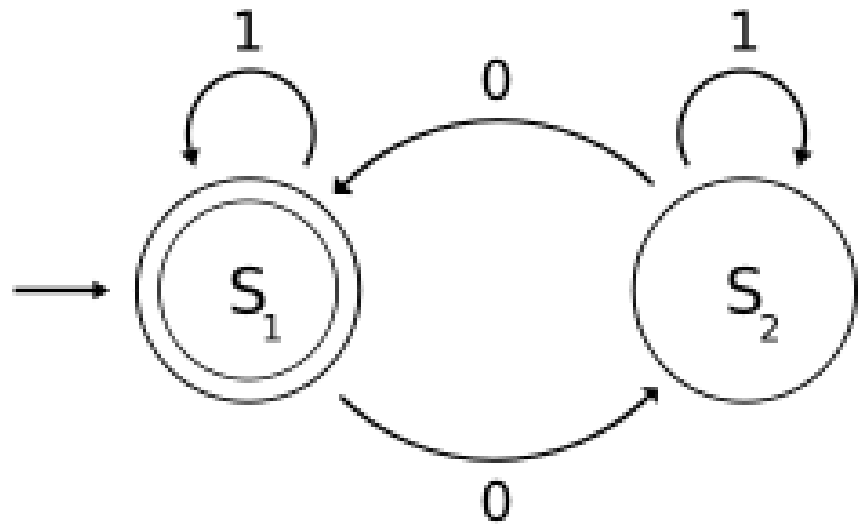
Accept	Not Accept
0101010	01111
1100	000
11010	00110



Accept되는 가장 짧은 String?


1
0
1
S1
S2
0

''



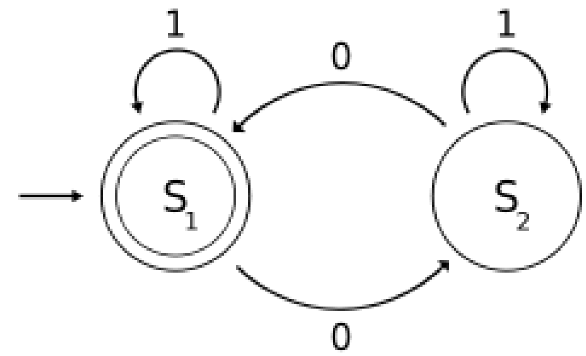
ε

$$(Q, \Sigma, \delta, q_0, A)$$

$$(Q, \Sigma, \delta, q_0, A)$$

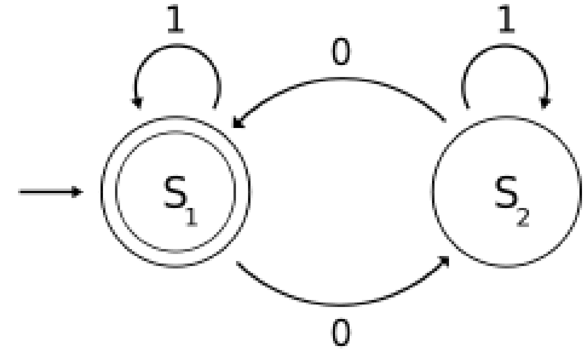
$$(Q, \Sigma, \delta, q_0, A)$$

a finite set of *states*.



$$(Q, \Sigma, \delta, q_0, A)$$

a finite set of *states*.



$(Q, \Sigma, \delta, q_0, A)$

a finite set of *states*.

$\{s_1, s_2\}$

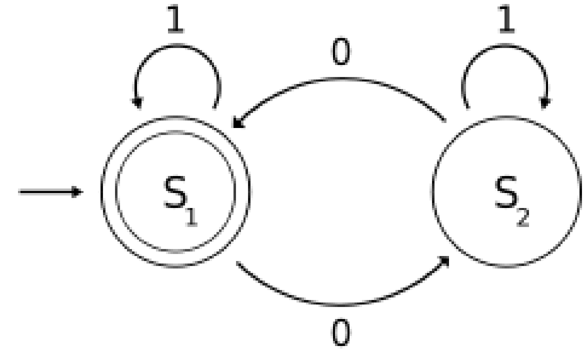
$$(Q, \Sigma, \delta, q_0, A)$$

$$(Q, \Sigma, \delta, q_0, A)$$

a finite set of *symbols*

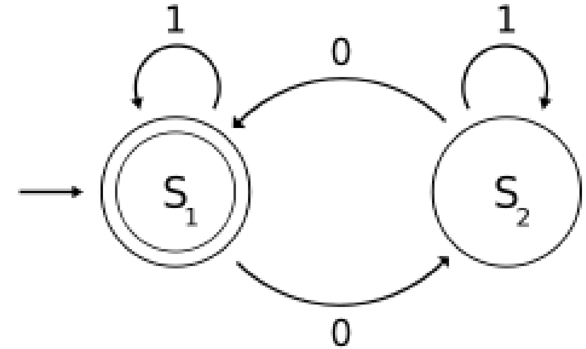
$$(Q, \Sigma, \delta, q_0, A)$$

a finite set of *symbols* : *alphabet*



$$(Q, \Sigma, \delta, q_0, A)$$

a finite set of *symbols* : *alphabet*



$(Q, \Sigma, \delta, q_0, A)$

a finite set of *symbols* : *alphabet*

$\{0, 1\}$

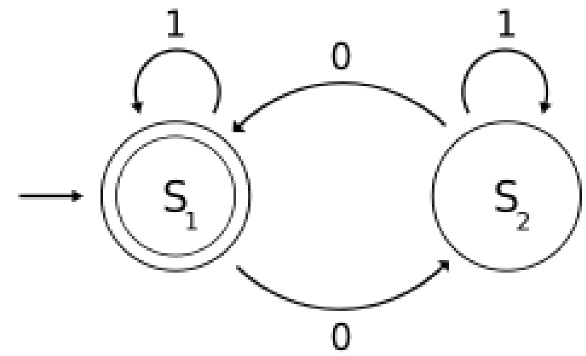
$$(Q, \Sigma, \delta, q_0, A)$$

$$(Q, \Sigma, \delta, q_0, A)$$

the *transition function*

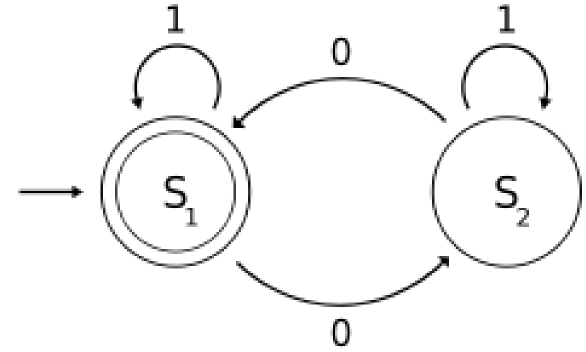
$$(Q, \Sigma, \delta, q_0, A)$$

the *transition function :* $\delta: Q \times \Sigma \rightarrow Q$



$(Q, \Sigma, \delta, q_0, A)$

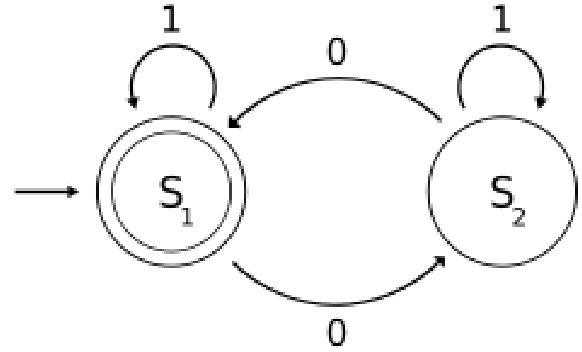
the *transition function :* $\delta: Q \times \Sigma \rightarrow Q$



$$(Q, \Sigma, \delta, q_0, A)$$

the *transition function* : $\delta: Q \times \Sigma \rightarrow Q$

$$\{ \ ((s_1, 0), s_2), ((s_1, 1), s_1), ((s_2, 0), s_1), ((s_2, 1), s_2) \ \}$$



$$(Q, \Sigma, \delta, q_0, A)$$

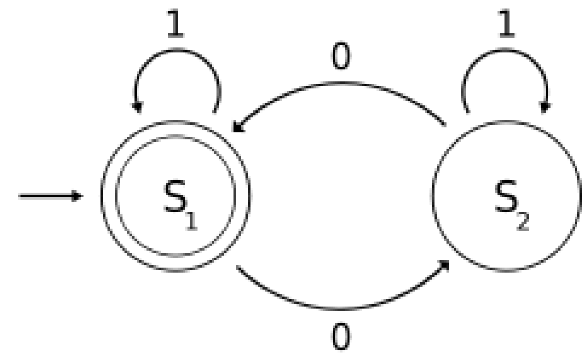
the *transition function* : $\delta: Q \times \Sigma \to Q$

	0	1
s_1	s_2	s_1
s_2	s_1	s_2

$$(Q, \Sigma, \delta, q_0, A)$$

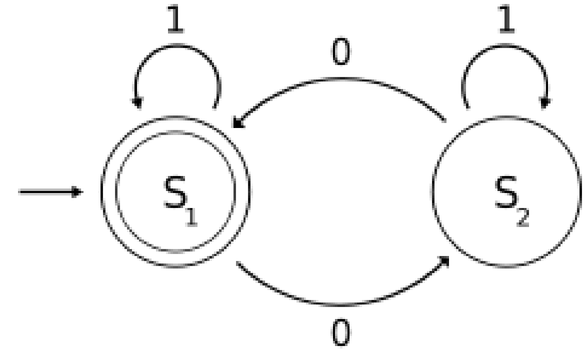
$$(Q, \Sigma, \delta, q_0, A)$$

the *start state*



$(Q, \Sigma, \delta, q_0, A)$

the *start state*



$(Q, \Sigma, \delta, q_0, A)$

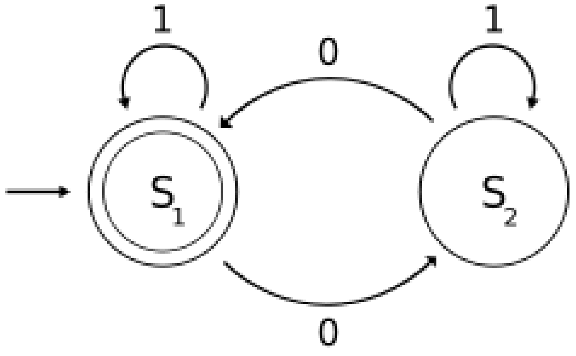
the *start state*

s_1

$$(Q, \Sigma, \delta, q_0, A)$$

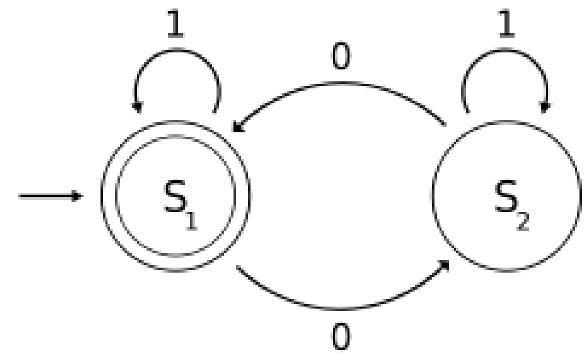
$$(Q, \Sigma, \delta, q_0, A)$$

accept states



$$(Q, \Sigma, \delta, q_0, A)$$

accept states

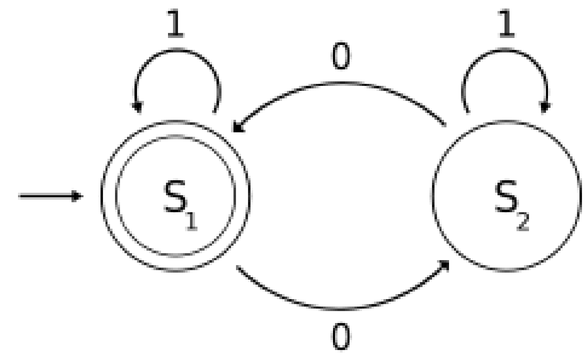


$$(Q, \Sigma, \delta, q_0, A)$$

accept states

$\{s_1\}$

$(Q, \Sigma, \delta, q_0, A)$



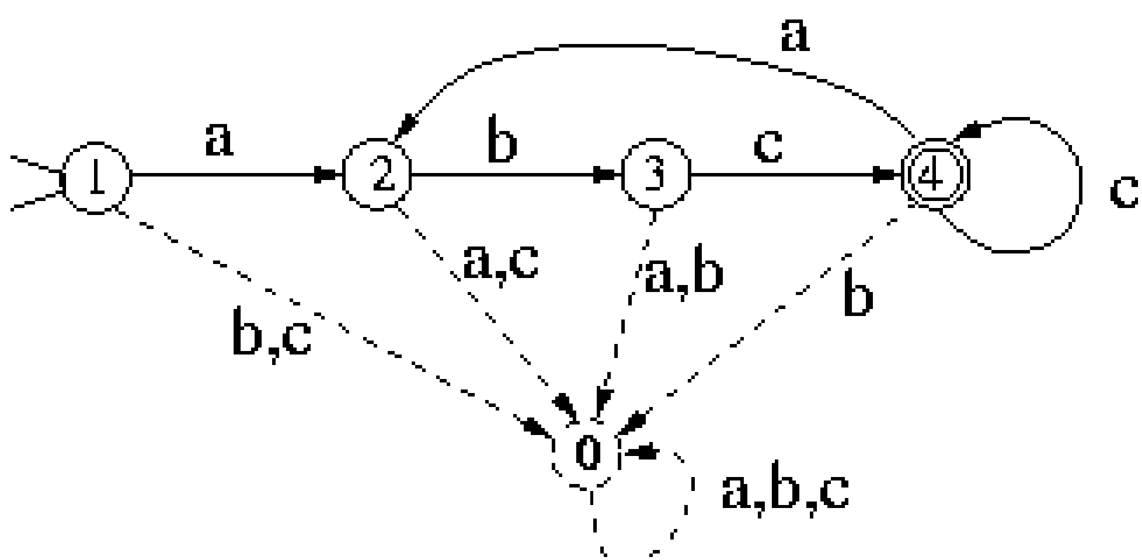
Q : $\{s_1, s_2\}$

Σ : $\{0, 1\}$

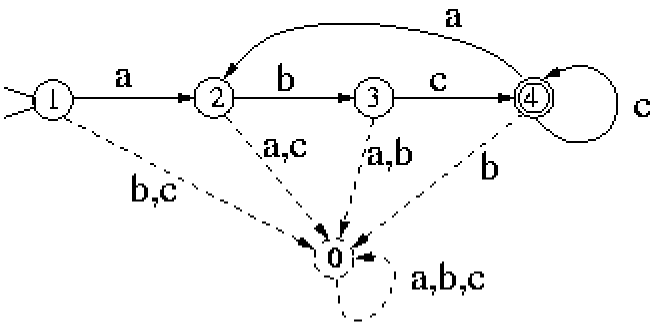
δ : $\{((s_1, 0), s_2), ((s_1, 1), s_1),$
$((s_2, 0), s_1), ((s_2, 1), s_2)\}$

q_0: s_1

A : $\{s_1\}$


a
a
b
c
c
1
2
3
4
a,c
a,b
b
b,c
0
a,b,c

$(Q, \Sigma, \delta, q_0, A)$



Q

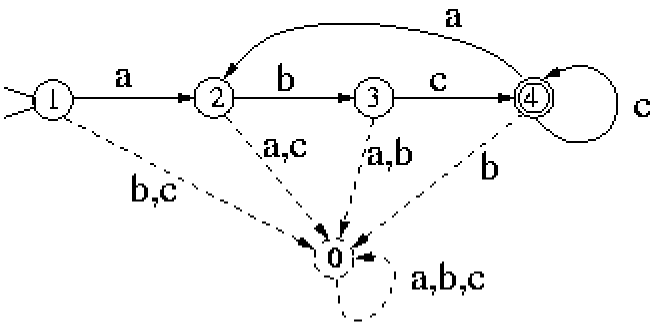
Σ

δ

q_0

A

$(Q, \Sigma, \delta, q_0, A)$



Q : {1, 2, 3, 4, 0}

Σ : {a, b, c}

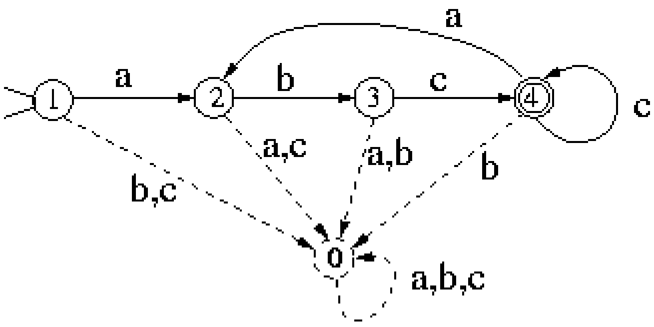
δ : { ((1, a), 2), ((1, b), 0), ((1, c), 0),
((2, a), 0), ((2, b), 3), ((2, c), 0),
((3, a), 0), ((3, b), 0), ((3, c), 4),
((4, a), 2), ((4, b), 0), ((4, c), 4) }

q_0 : 1

A : {4}

?

$(Q, \Sigma, \delta, q_0, A)$



Q : {1, 2, 3, 4, 0}

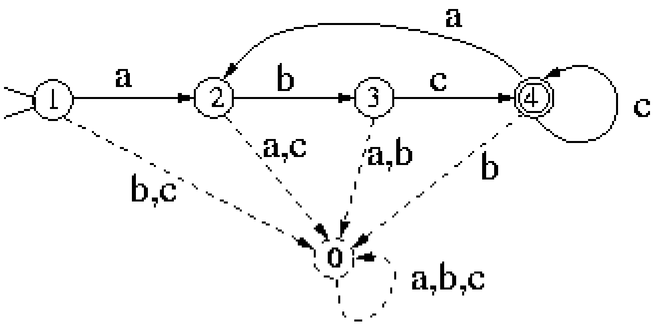
Σ : {a, b, c}

δ : { ((1, a), 2), ((1, b), 0), ((1, c), 0),
((2, a), 0), ((2, b), 3), ((2, c), 0),
((3, a), 0), ((3, b), 0), ((3, c), 4),
((4, a), 2), ((4, b), 0), ((4, c), 4),
((0, a), 0), ((0, b), 0), ((0, c), 0) }

q_0 : 1

A : {4}

$(Q, \Sigma, \delta, q_0, A)$

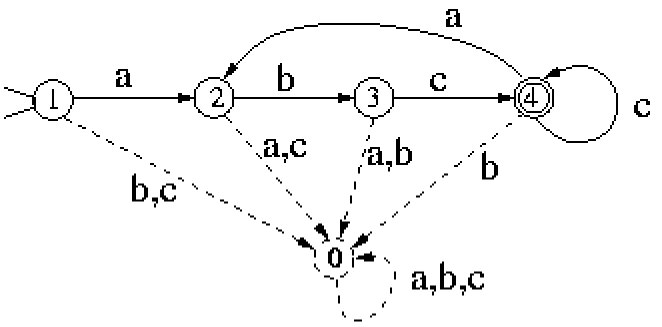


Q : {1, 2, 3, 4, 0}
Σ : {a, b, c}
δ :

	a	b	c
1	2	0	0
2	0	3	0
3	0	0	4
4	2	0	4
0	0	0	0

q_0 : 1
A : {4}

$(Q, \Sigma, \delta, q_0, A)$



Q : {1, 2, 3, 4, 0}

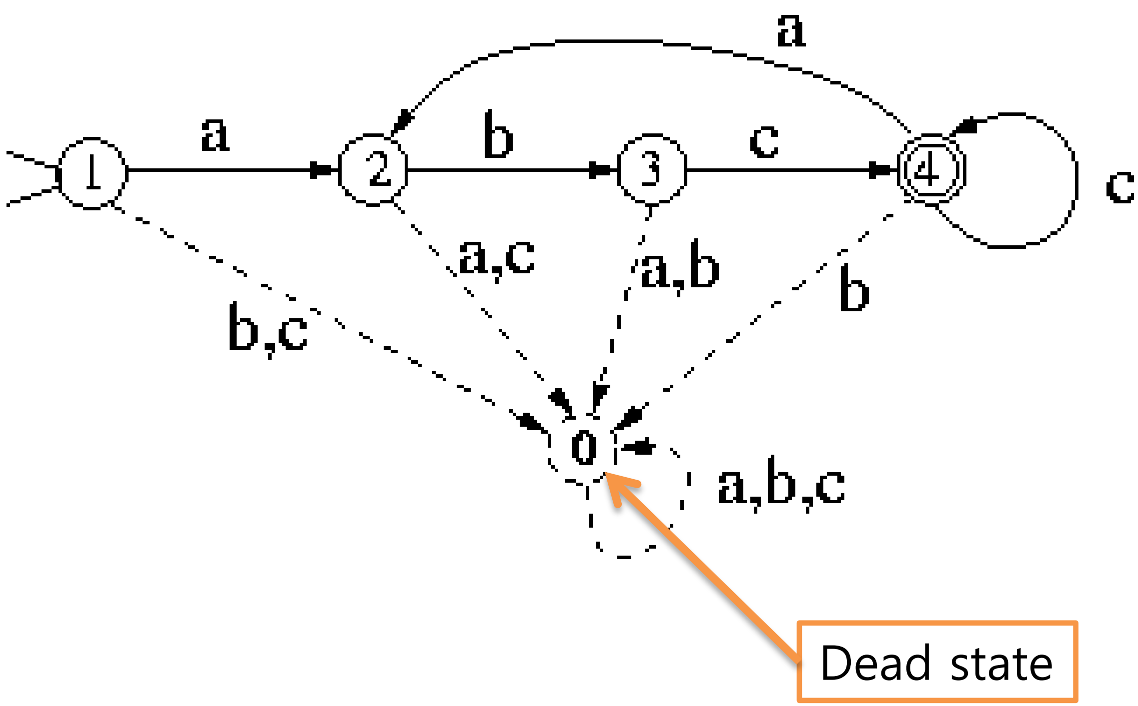
Σ : {a, b, c}

δ :

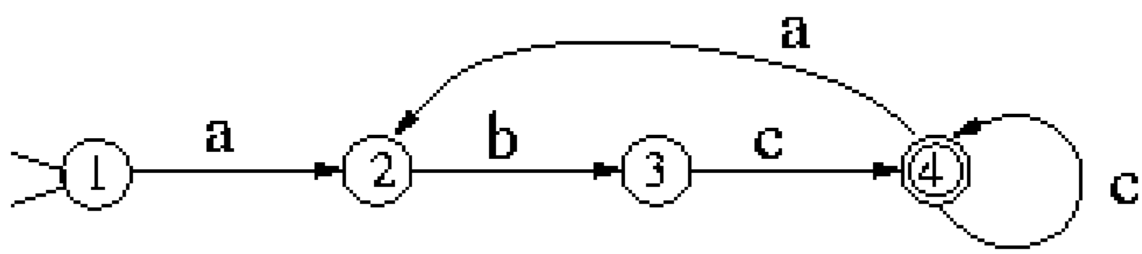
	a	b	c
1	2		
2		3	
3			4
4	2		4

q_0 : 1

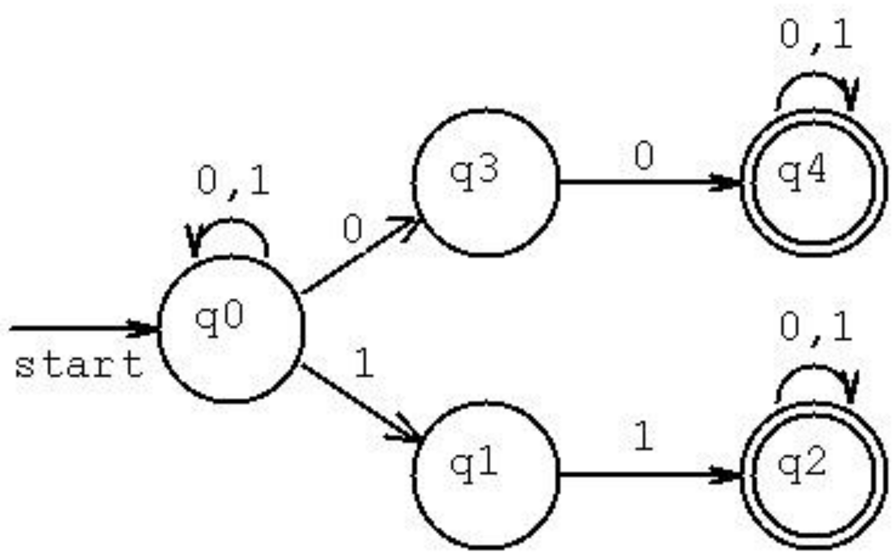
A : {4}



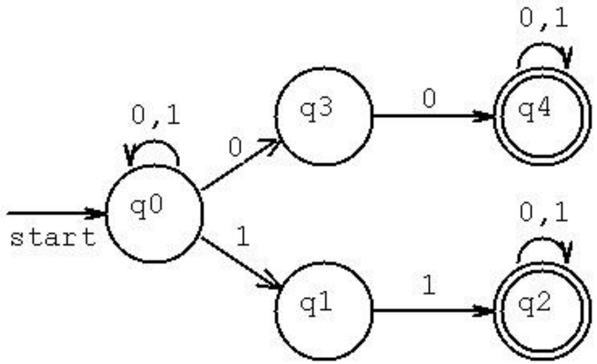
a
a
b
c
c
1
2
3
4
0
b,c
a,c
a,b
b
a,b,c
Dead state



1
2
3
4
a
b
c
a
c

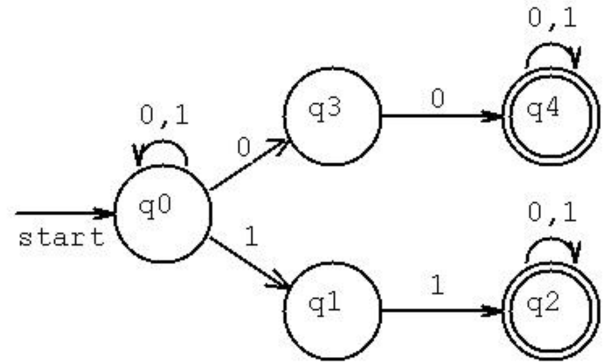

0,1
0,1
0
q3
0
q4
q0
start
0,1
1
q1
1
q2

$(Q, \Sigma, \delta, q_0, A)$



Q : $\{q_0, q_1, q_2, q_3, q_4\}$
Σ : $\{0, 1\}$
δ : $\{ ((q_0, 0), q_0), ((q_0, 1), q_0),$
$((q_0, 0), q_3), ((q_0, 1), q_1),$
$((q_3, 0), q_4), ((q_1, 1), q_2),$
$((q_4, 0), q_4), ((q_4, 1), q_4),$
$((q_2, 0), q_2), ((q_2, 1), q_2) \}$
q_0 : q_0
A : $\{q_2, q_4\}$

$(Q, \Sigma, \delta, q_0, A)$



Q : $\{q_0, q_1, q_2, q_3, q_4\}$

Σ : $\{0, 1\}$

δ :

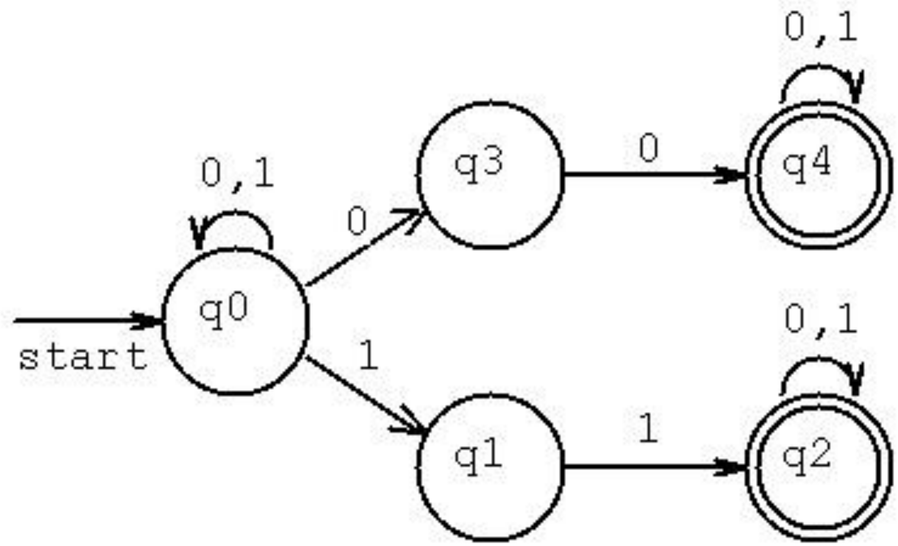
	0	1
q_0	q_0, q_3	q_0, q_1
q_1		q_2
q_2	q_2	q_2
q_3	q_4	
q_4	q_4	q_4

q_0 : q_0

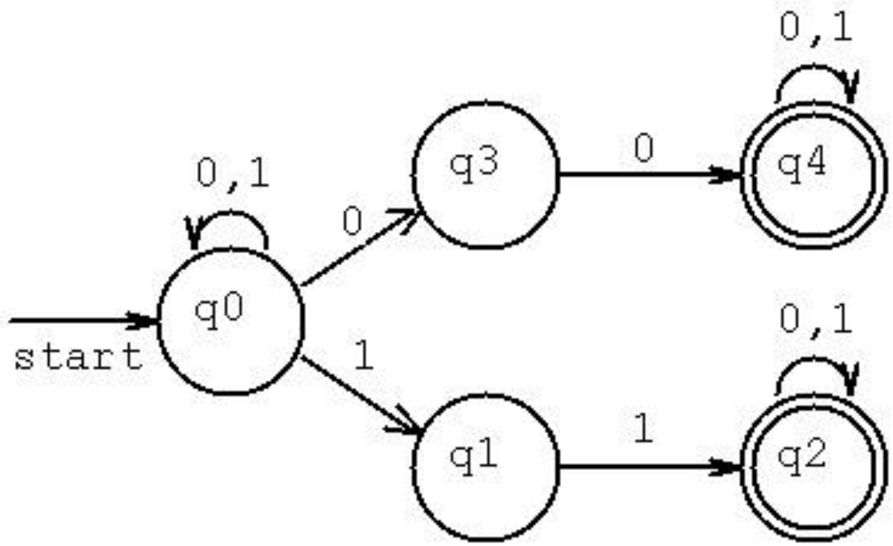
A : $\{q_2, q_4\}$

$$(Q, \Sigma, \delta, q_0, A)$$

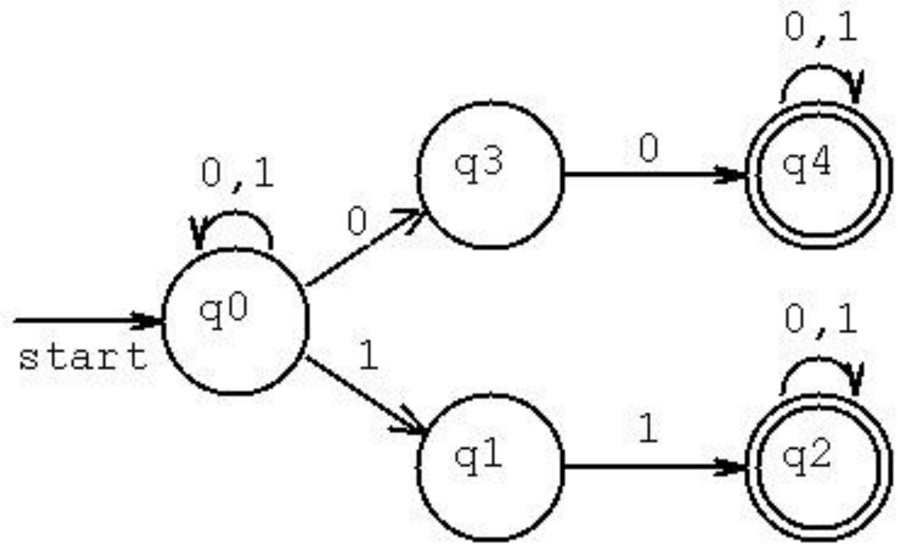
the *transition function* : $\delta: Q \times \Sigma \rightarrow Q$



the *transition function :* δ: Q × Σ → Q



the *transition function :* $\delta: Q \times \Sigma \rightarrow Q$



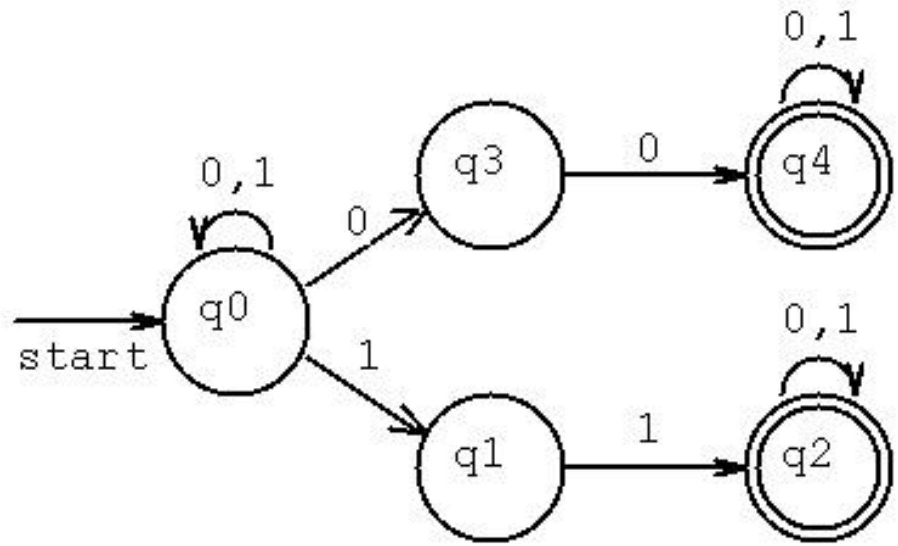
the *transition function :* δ: Q × Σ → Q

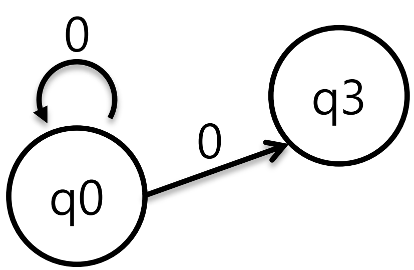
Q × Σ :

{ $(q_0, 0)$, $(q_0, 1)$,
$(q_1, 0)$, $(q_1, 1)$,
$(q_2, 0)$, $(q_2, 1)$,
$(q_3, 0)$, $(q_3, 1)$,
$(q_4, 0)$, $(q_4, 1)$ }

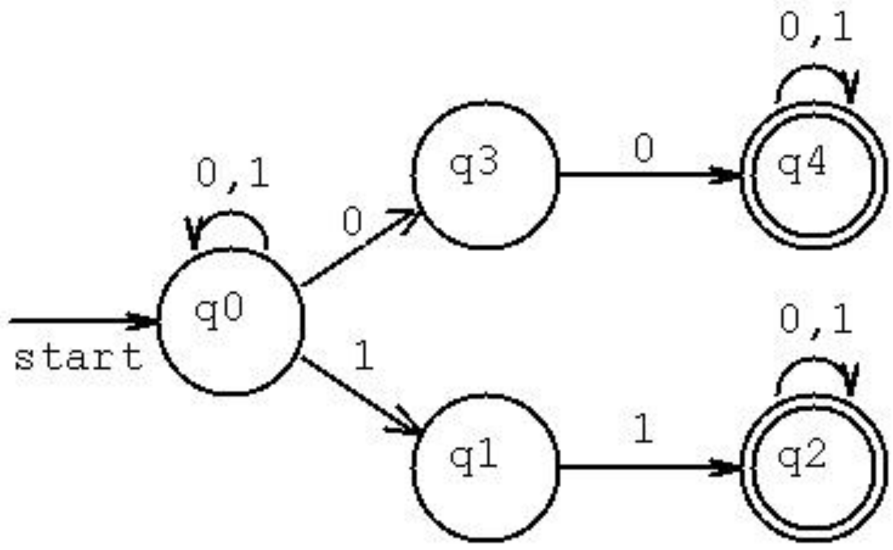
δ :

{ $((q_0, 0), q_0)$, $((q_0, 1), q_0)$,
$((q_0, 0), q_3)$, $((q_0, 1), q_1)$,
$((q_3, 0), q_4)$, $((q_1, 1), q_2)$,
$((q_4, 0), q_4)$, $((q_4, 1), q_4)$,
$((q_2, 0), q_2)$, $((q_2, 1), q_2)$ }

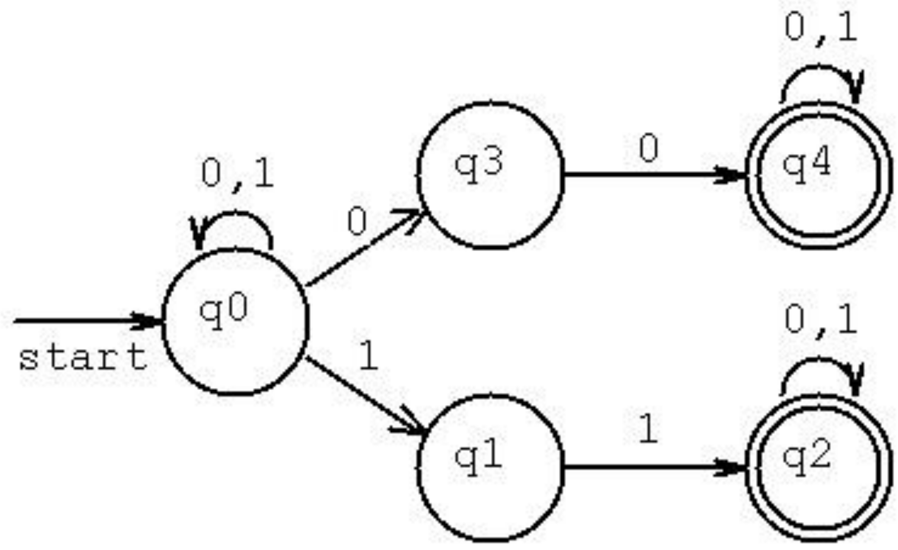




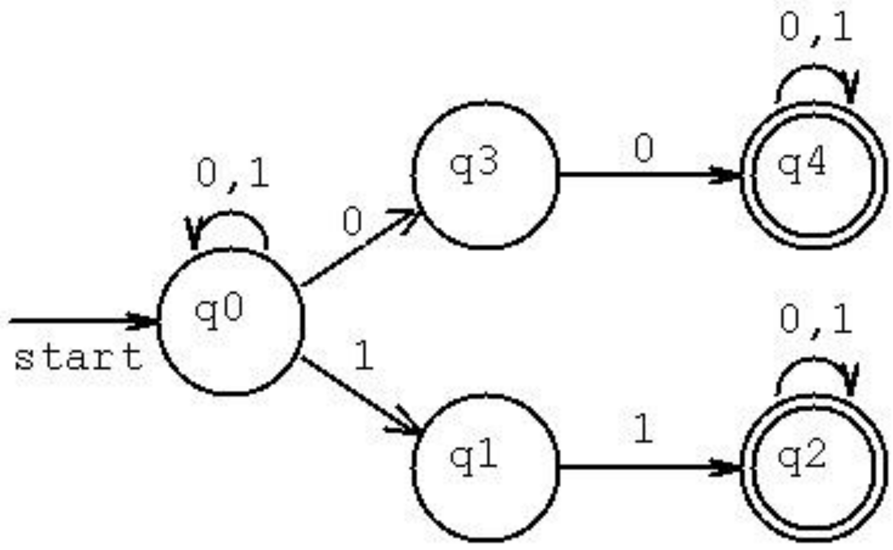
- 둘 다 가능!
- 두 개의 경로로 다 해본다.
- 하나라도 accept되면 accept.



'011' : q0 → q3 Not Accept

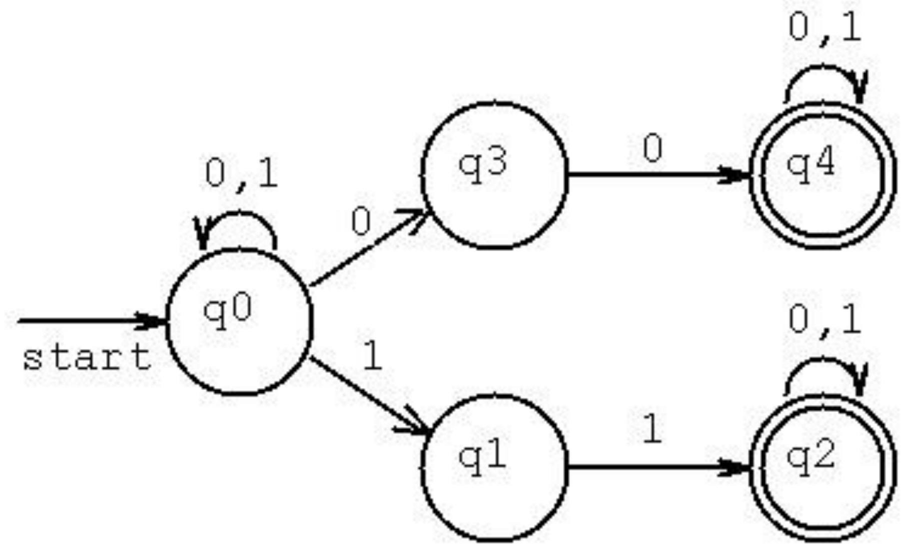


‘011’ : q0 → q0 → q1 → q2

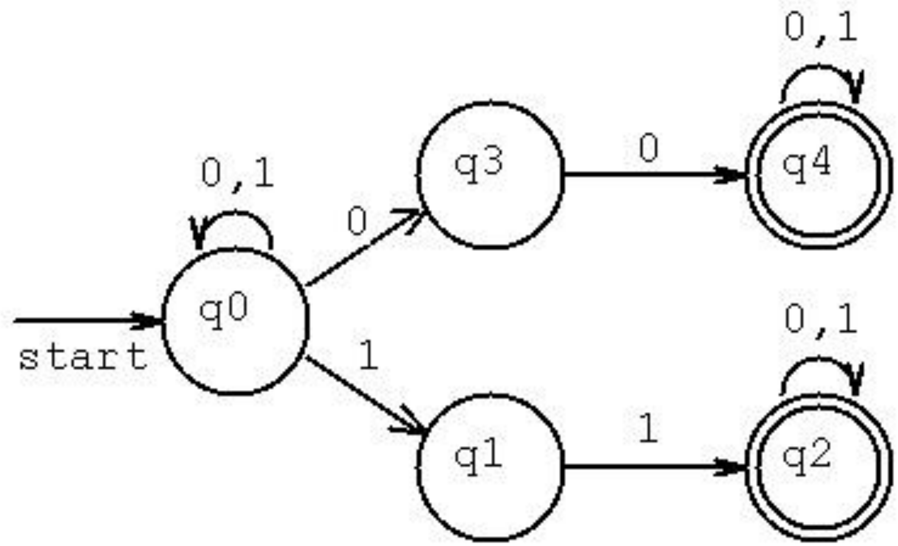


'001' : q0 → q0 → q0 → q3

Not Accept

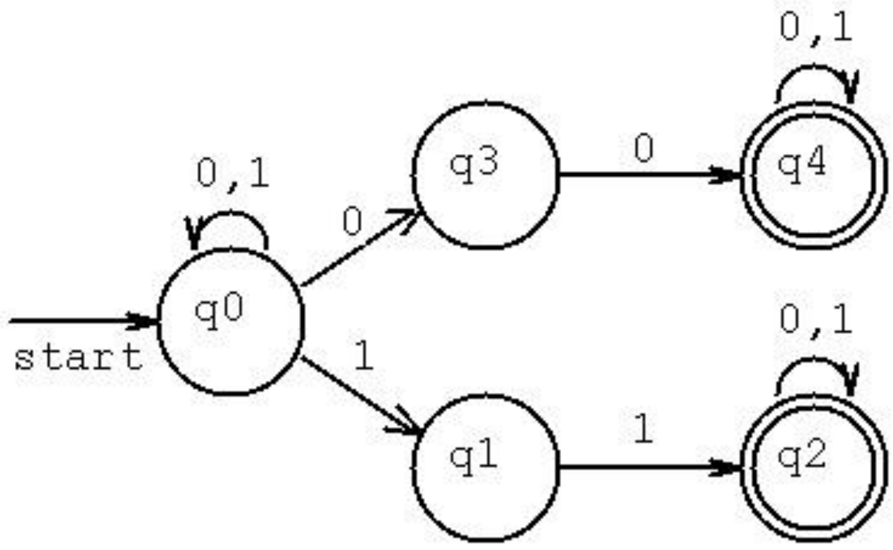


'001' : q0 → q3 → q4 → q4



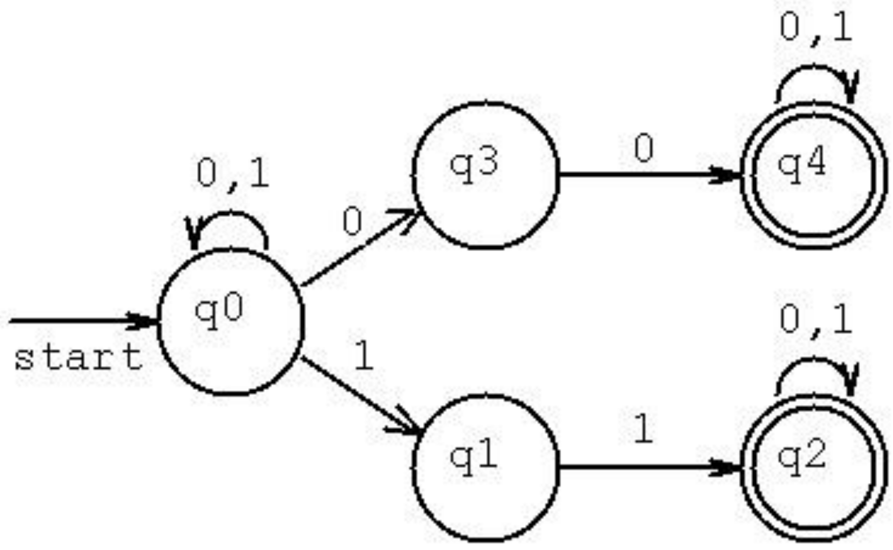
'001' : q0 → q3 → q4 → q4

'011' : q0 → q0 → q1 → q2



‘001’ : q0 → q3 → q4 → q4

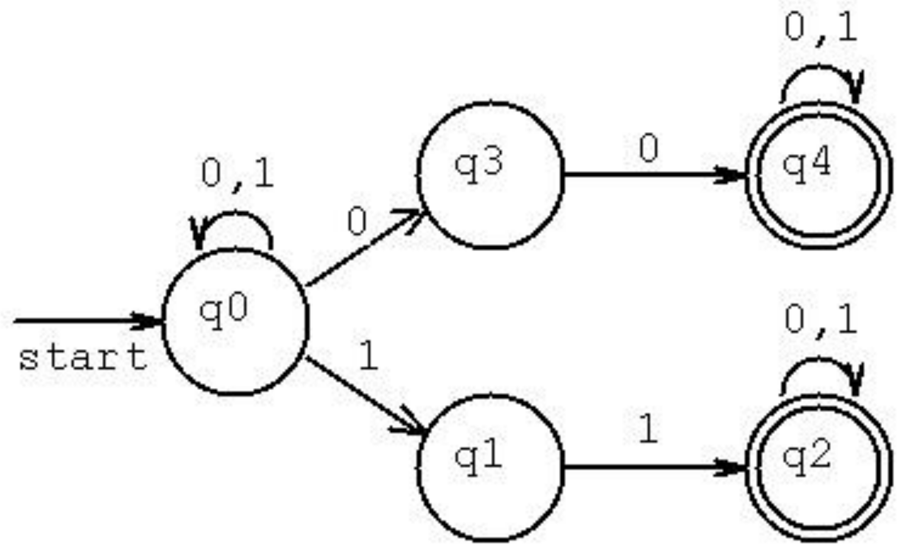
‘011’ : q0 → q0 → q1 → q2



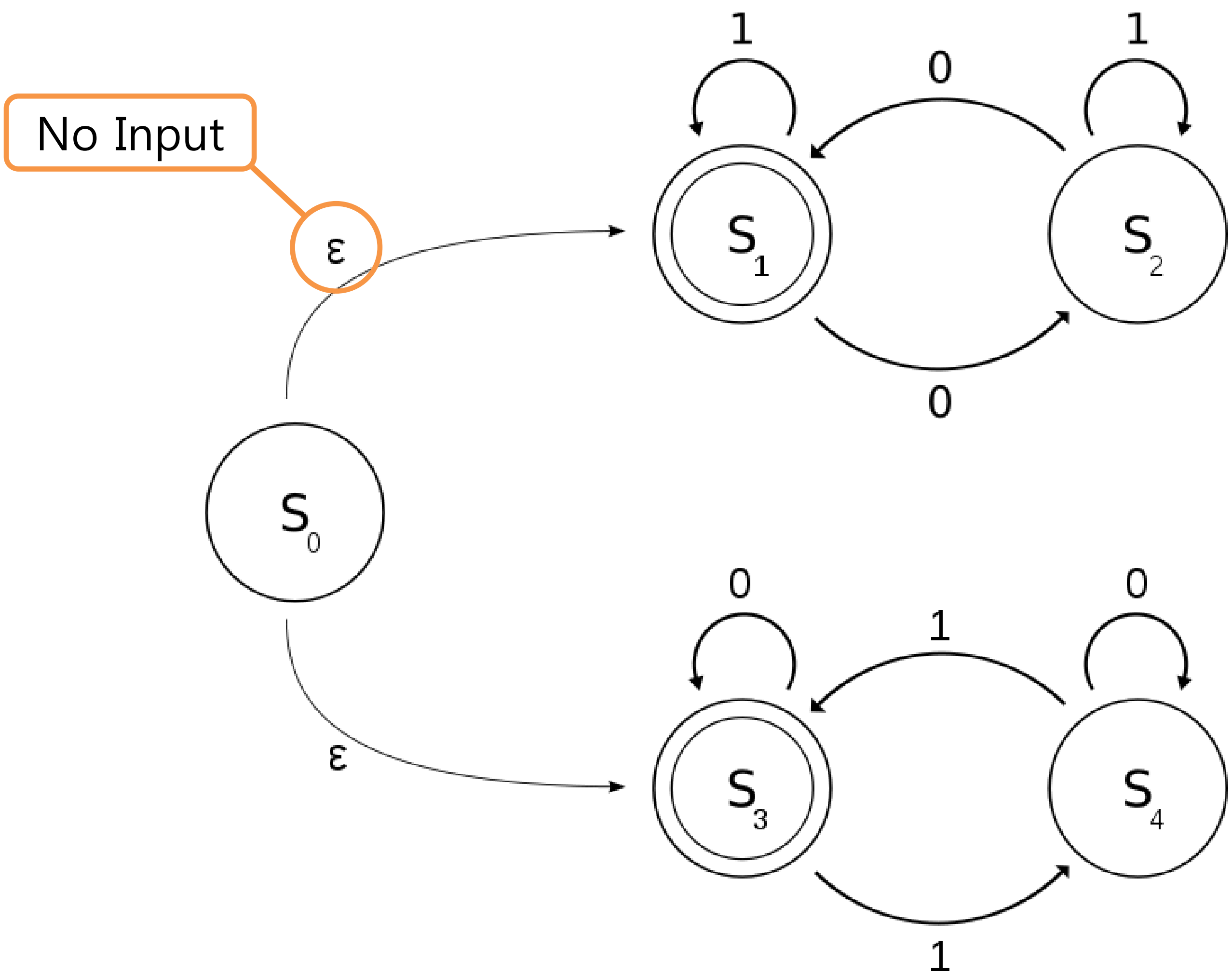
‘001’ : q0 → q3 → q4 → q4

‘011’ : q0 → q0 → q1 → q2

Nondeterministic



Nondeterministic Finite Automata


No Input
ε
1
0
1
S1
S2
0
S0
0
1
0
S3
S4
1
ε

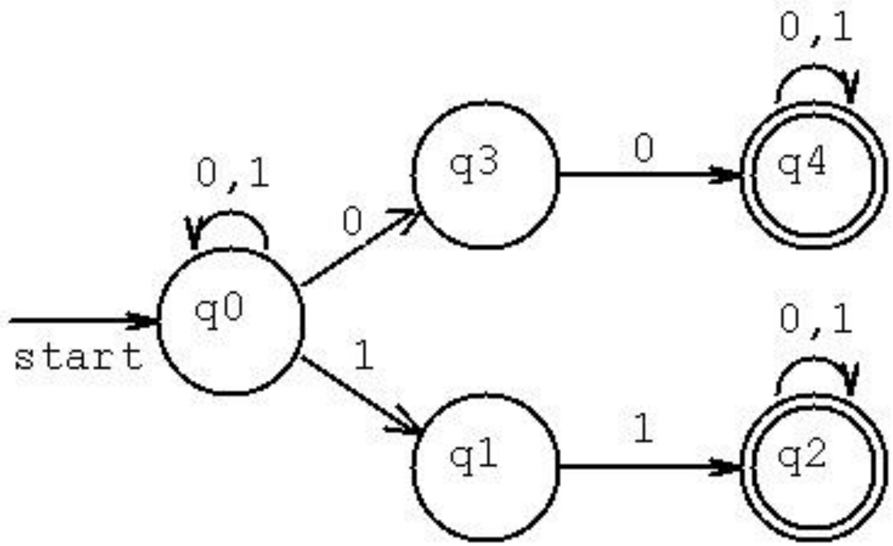
NFA

DFA

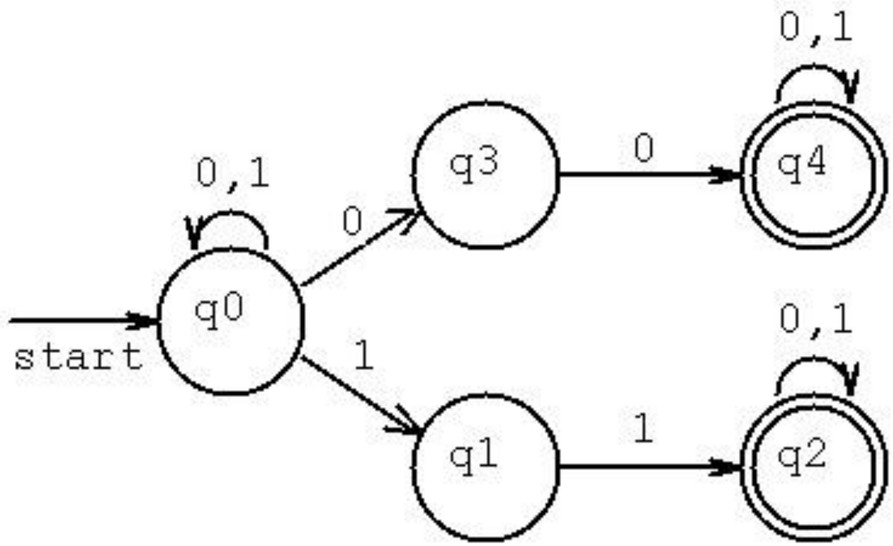
Deterministic Finite Automata

NFA $\supset$ DFA

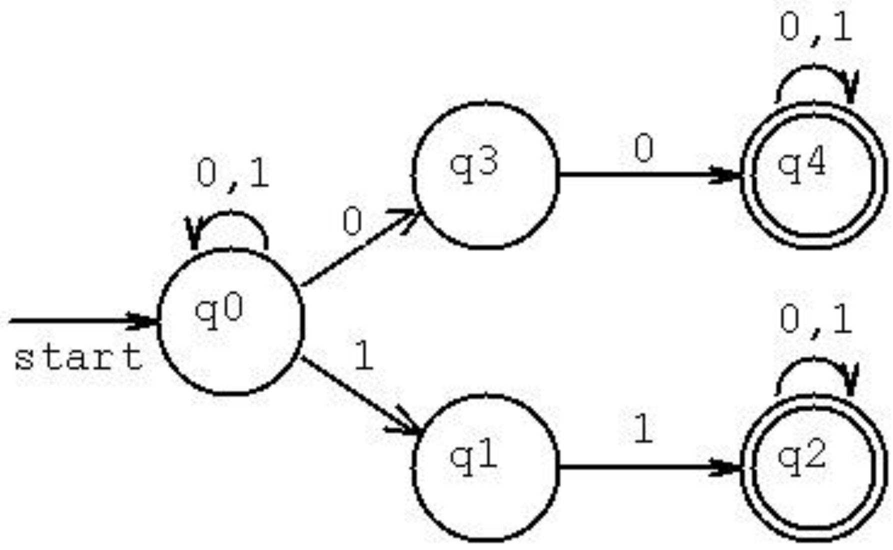
NFA ⇒ DFA



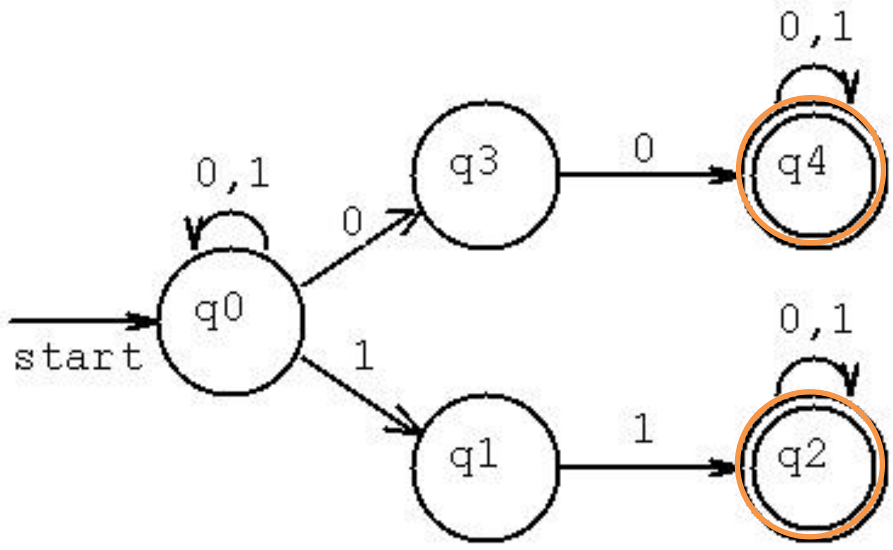
Nondeterministic Finite Automata



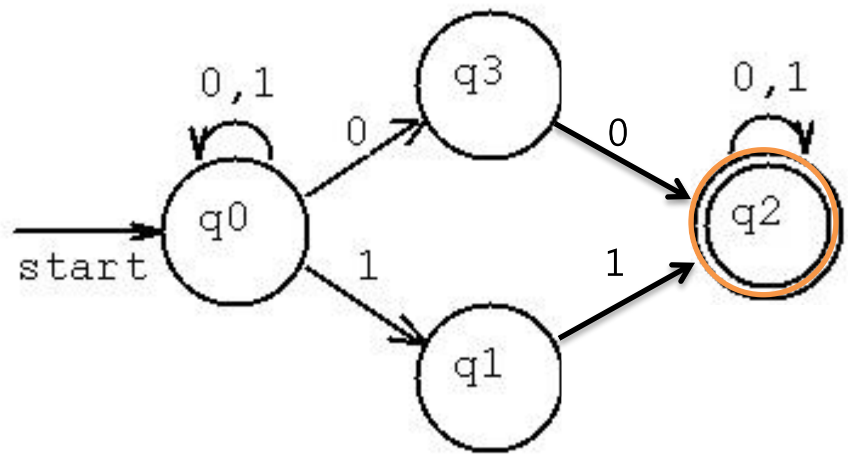
Nondeterministic Finite Automata
⇒ Deterministic Finite Automata



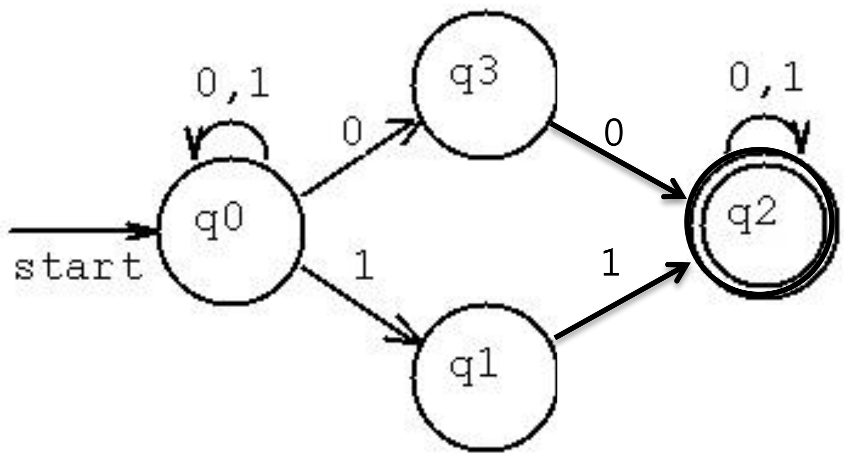
	0	1
q_0	q_0, q_3	q_0, q_1
q_1		q_2
q_2	q_2	q_2
q_3	q_4	
q_4	q_4	q_4



	0	1
q_0	q_0, q_3	q_0, q_1
q_1		q_2
q_2	q_2	q_2
q_3	q_4	
q_4	q_4	q_4

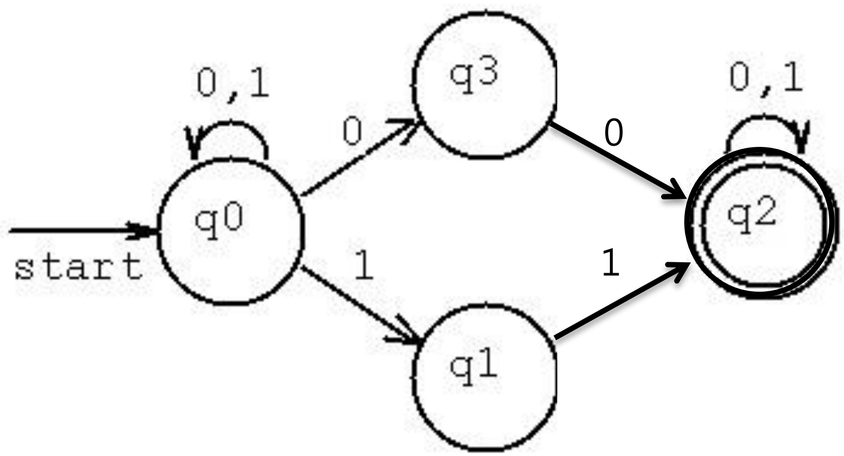


	0	1
q_0	q_0, q_3	q_0, q_1
q_1		q_2
q_2	q_2	q_2
q_3	q_2	



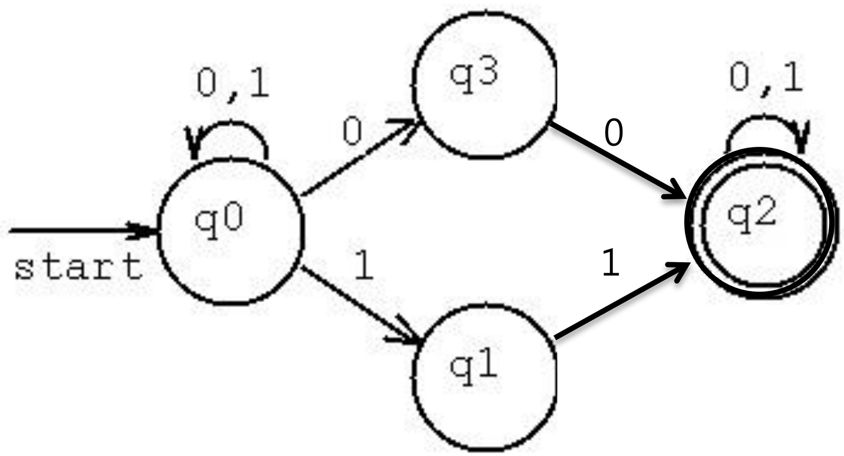
	0	1
q_0	q_0, q_3	q_0, q_1
q_1		q_2
q_2	q_2	q_2
q_3	q_2	

	0	1
q_0		



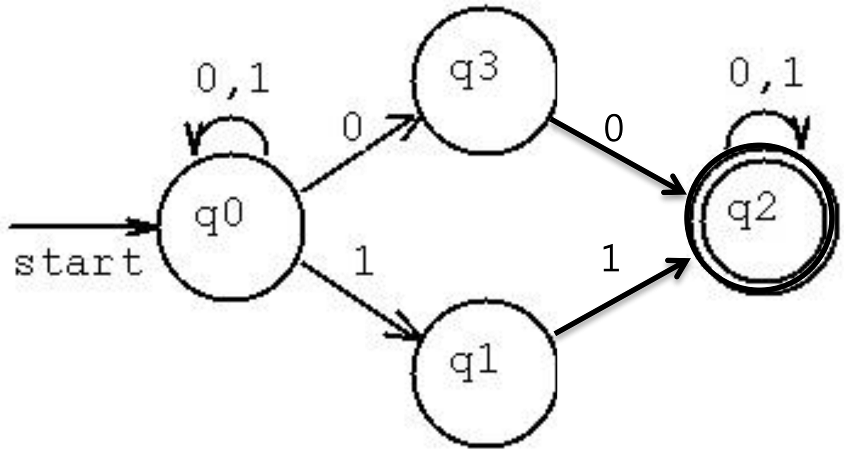
	0	1
q_0	q_0, q_3	q_0, q_1
q_1		q_2
q_2	q_2	q_2
q_3	q_2	

	0	1
q_0	q_0, q_3	q_0, q_1



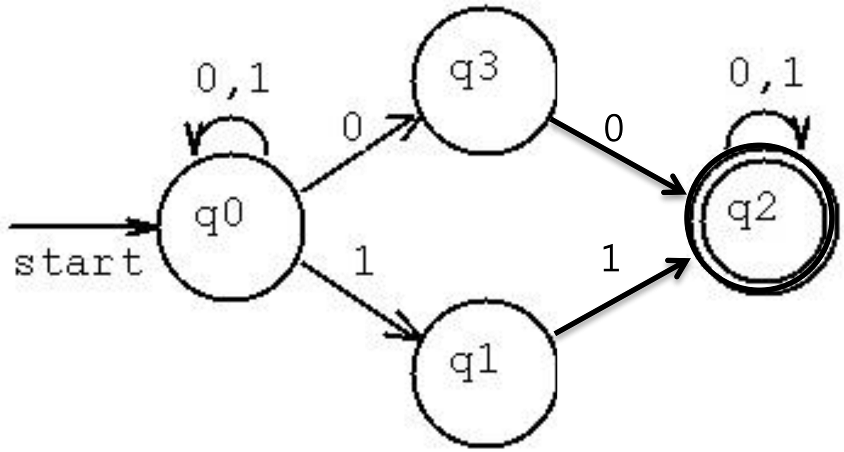
	0	1
q_0	q_0, q_3	q_0, q_1
q_1		q_2
q_2	q_2	q_2
q_3	q_2	

	0	1
q_0	q_0, q_3	q_0, q_1
q_0, q_3		
q_0, q_1		



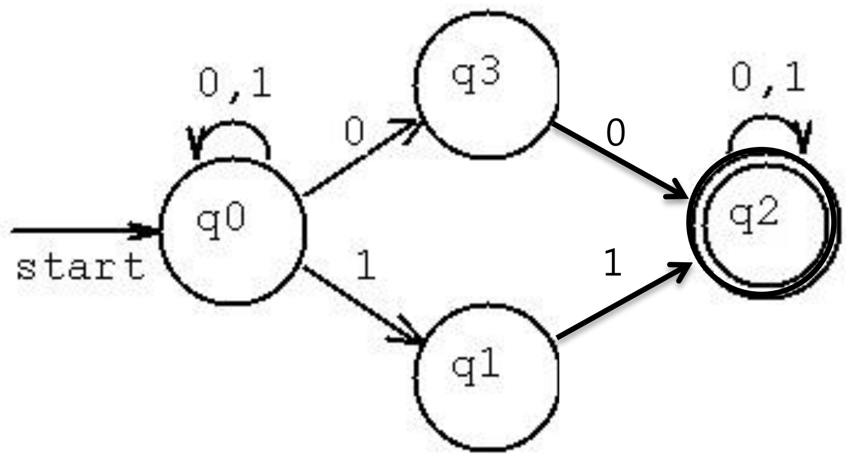
	0	1
q_0	q_0, q_3	q_0, q_1
q_1		q_2
q_2	q_2	q_2
q_3	q_2	

	0	1
q_0	q_0, q_3	q_0, q_1
q_0, q_3	q_0, q_2, q_3	
q_0, q_1		



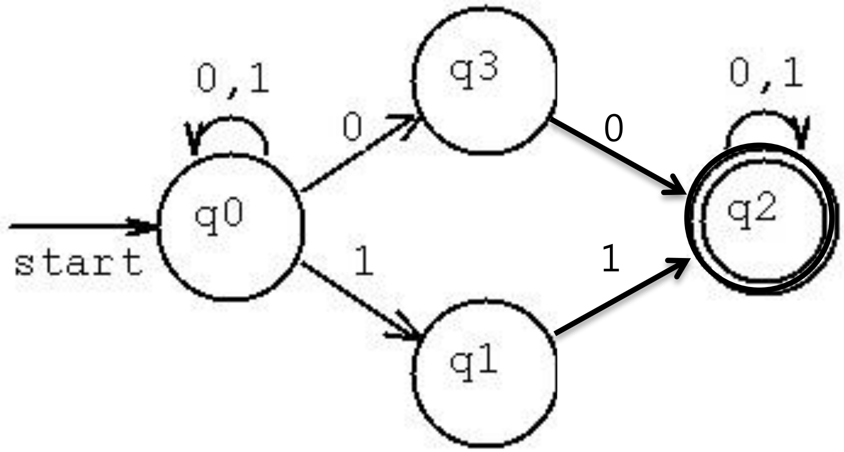
	0	1
q_0	q_0, q_3	q_0, q_1
q_1		q_2
q_2	q_2	q_2
q_3	q_2	

	0	1
q_0	q_0, q_3	q_0, q_1
q_0, q_3	q_0, q_2, q_3	q_0, q_1
q_0, q_1		



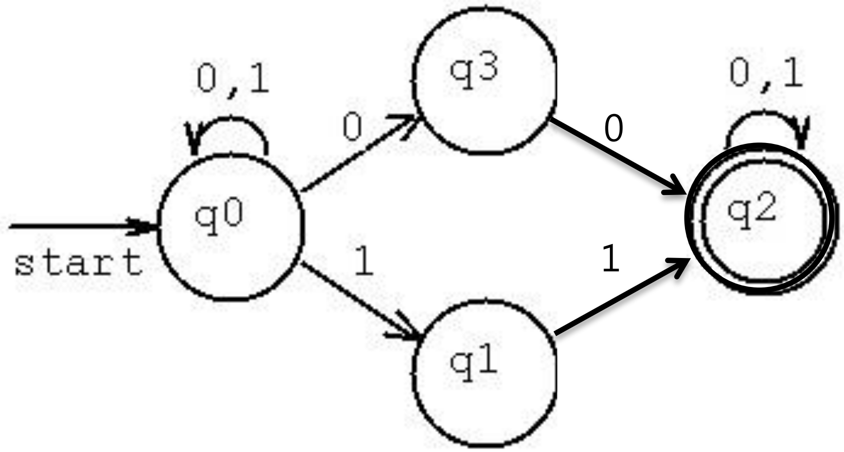
	0	1
q_0	q_0, q_3	q_0, q_1
q_1		q_2
q_2	q_2	q_2
q_3	q_2	

	0	1
q_0	q_0, q_3	q_0, q_1
q_0, q_3	q_0, q_2, q_3	q_0, q_1
q_0, q_1	q_0, q_3	



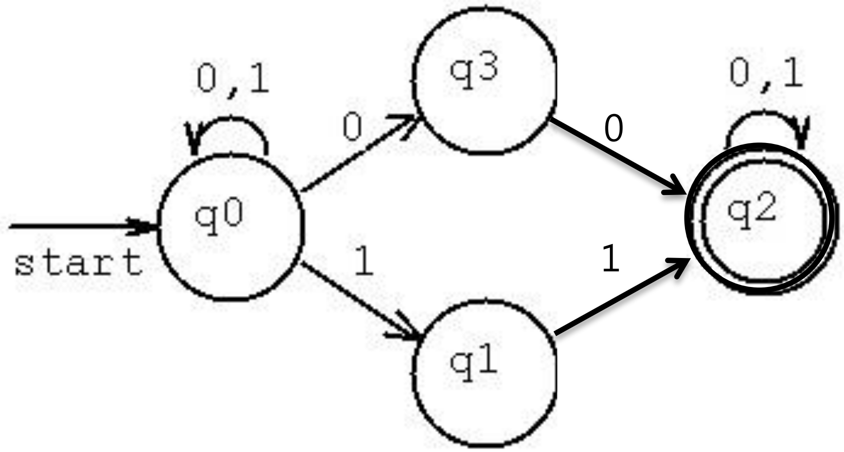
	0	1
q_0	q_0, q_3	q_0, q_1
q_1		q_2
q_2	q_2	q_2
q_3	q_2	

	0	1
q_0	q_0, q_3	q_0, q_1
q_0, q_3	q_0, q_2, q_3	q_0, q_1
q_0, q_1	q_0, q_3	q_0, q_1, q_2



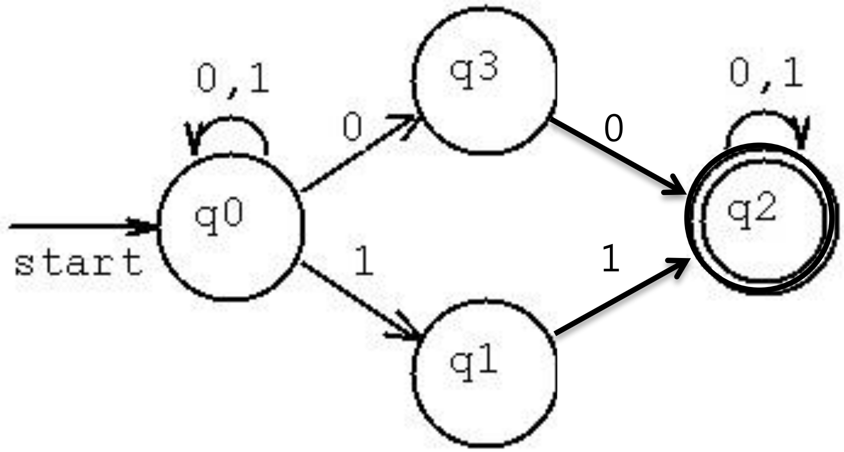
	0	1
q_0	q_0, q_3	q_0, q_1
q_1		q_2
q_2	q_2	q_2
q_3	q_2	

	0	1
q_0	q_0, q_3	q_0, q_1
q_0, q_3	q_0, q_2, q_3	q_0, q_1
q_0, q_1	q_0, q_3	q_0, q_1, q_2
q_0, q_2, q_3		
q_0, q_1, q_2		



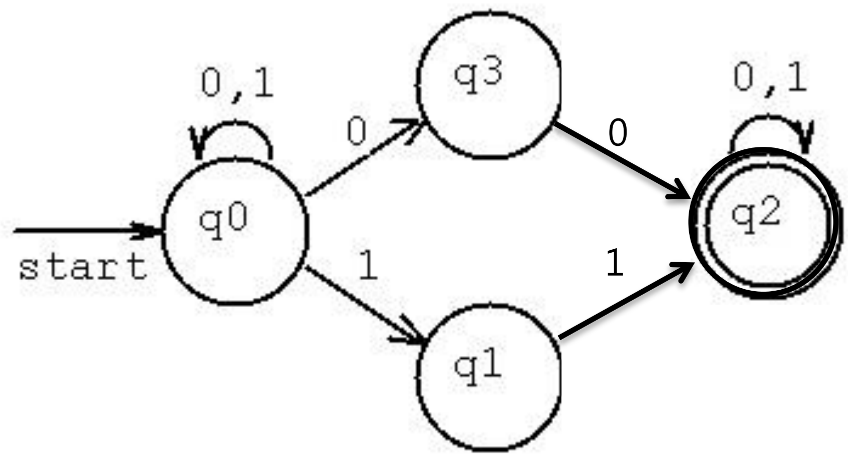
	0	1
q_0	q_0, q_3	q_0, q_1
q_1		q_2
q_2	q_2	q_2
q_3	q_2	

	0	1
q_0	q_0, q_3	q_0, q_1
q_0, q_3	q_0, q_2, q_3	q_0, q_1
q_0, q_1	q_0, q_3	q_0, q_1, q_2
q_0, q_2, q_3	q_0, q_2, q_3	q_0, q_1, q_2
q_0, q_1, q_2		



	0	1
q_0	q_0, q_3	q_0, q_1
q_1		q_2
q_2	q_2	q_2
q_3	q_2	

	0	1
q_0	q_0, q_3	q_0, q_1
q_0, q_3	q_0, q_2, q_3	q_0, q_1
q_0, q_1	q_0, q_3	q_0, q_1, q_2
q_0, q_2, q_3	q_0, q_2, q_3	q_0, q_1, q_2
q_0, q_1, q_2	q_0, q_2, q_3	q_0, q_1, q_2



	0	1
q_0	q_0, q_3	q_0, q_1
q_1		q_2
q_2	q_2	q_2
q_3	q_2	

	0	1
q_0	q_0, q_3	q_0, q_1
q_0, q_3	q_0, q_2, q_3	q_0, q_1
q_0, q_1	q_0, q_3	q_0, q_1, q_2
q_0, q_2, q_3	q_0, q_2, q_3	q_0, q_1, q_2
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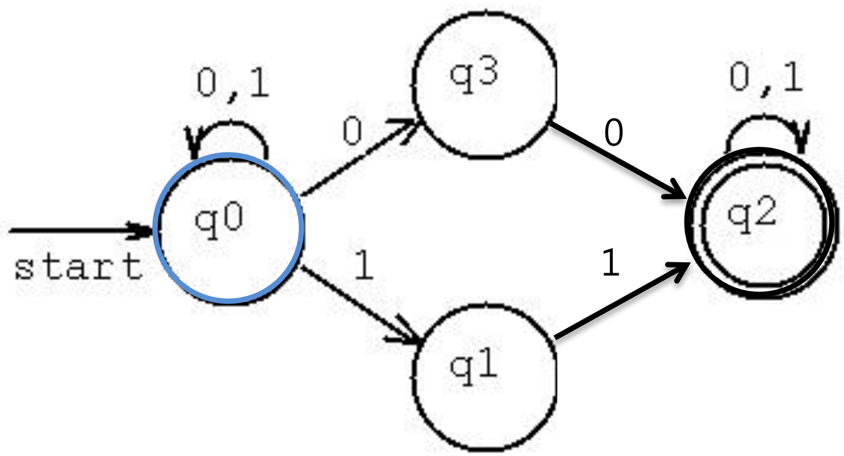

	0	1
q_0	q_0, q_3	q_0, q_1
q_1		q_2
q_2	q_2	q_2
q_3	q_2	

	0	1
q_0	q_0, q_3	q_0, q_1
q_0, q_3	q_0, q_2, q_3	q_0, q_1
q_0, q_1	q_0, q_3	q_0, q_1, q_2
q_0, q_2, q_3	q_0, q_2, q_3	q_0, q_1, q_2
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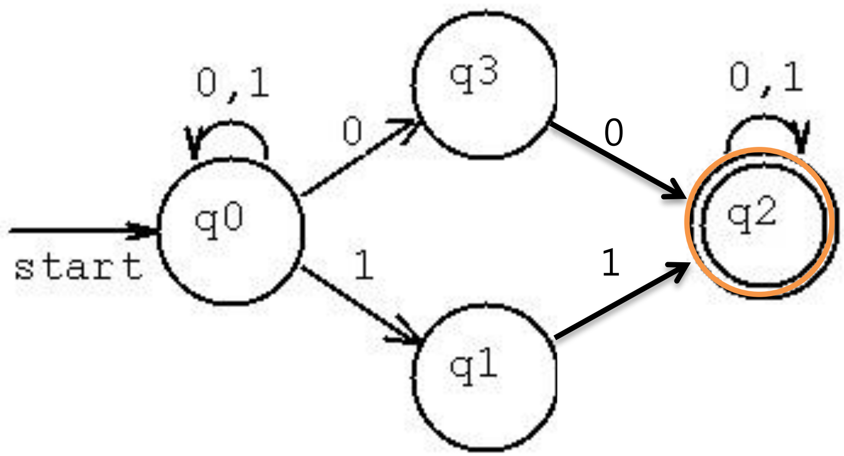

	0	1
q_0	q_0, q_3	q_0, q_1
q_1		q_2
q_2	q_2	q_2
q_3	q_2	

	0	1
q_0	q_0, q_3	q_0, q_1
q_0, q_3	q_0, q_2, q_3	q_0, q_1
q_0, q_1	q_0, q_3	q_0, q_1, q_2
q_0, q_2, q_3	q_0, q_2, q_3	q_0, q_1, q_2
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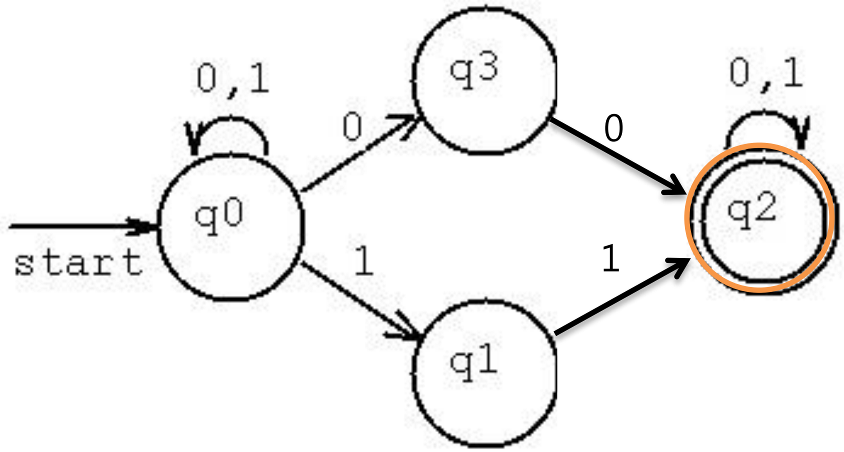
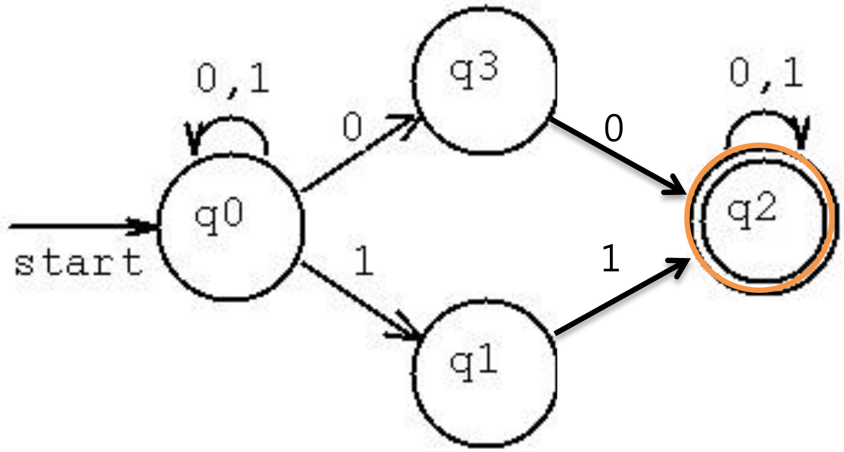


	0	1
q_0	q_0, q_3	q_0, q_1
q_1		q_2
q_2	q_2	q_2
q_3	q_2	

	0	1
q_0	q_0, q_3	q_0, q_1
q_0, q_3	q_0, q_2, q_3	q_0, q_1
q_0, q_1	q_0, q_3	q_0, q_1, q_2
q_0, q_2, q_3	q_0, q_2, q_3	q_0, q_1, q_2
q_0, q_1, q_2	q_0, q_2, q_3	q_0, q_1, q_2

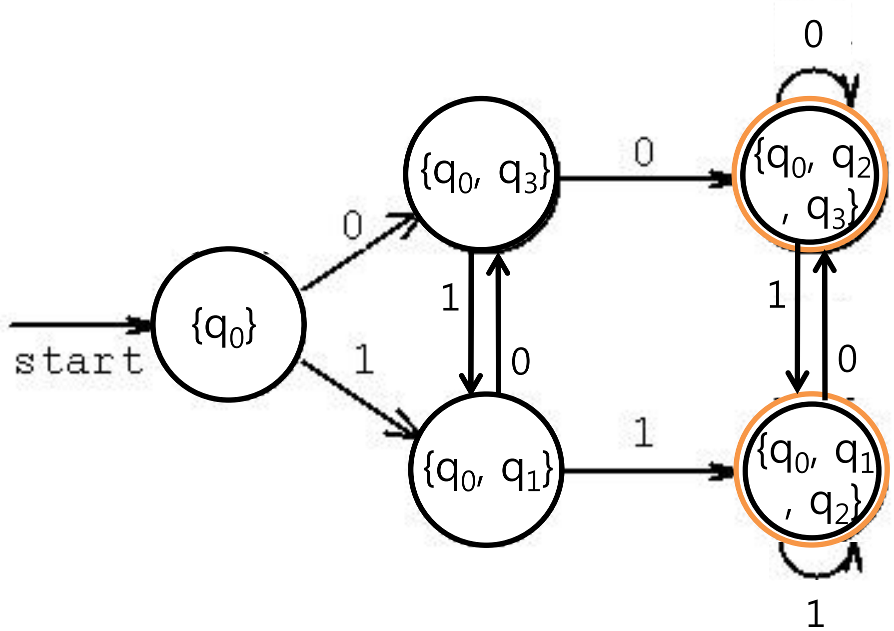


	0	1
q_0	q_0, q_3	q_0, q_1
q_1		q_2
q_2	q_2	q_2
q_3	q_2	

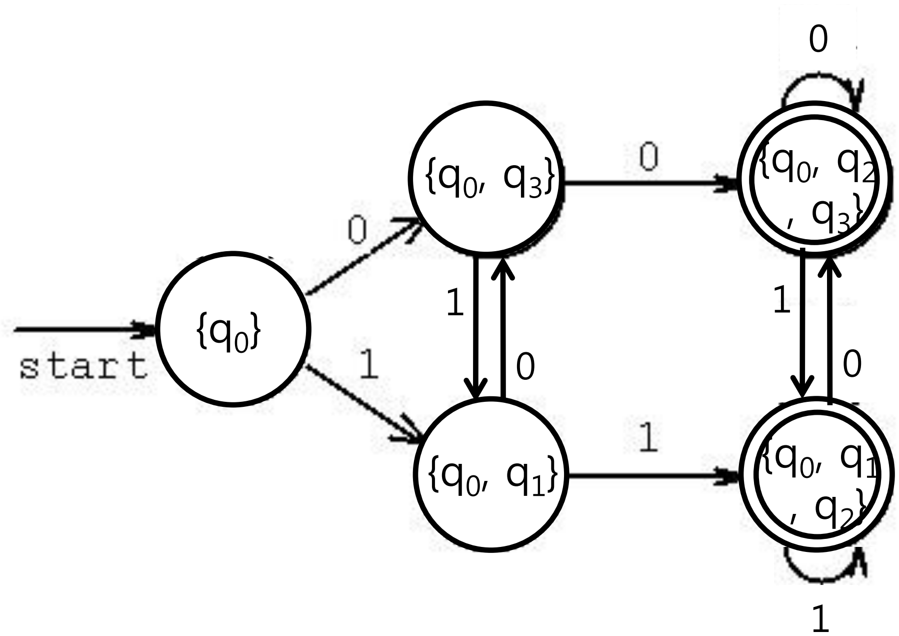
	0	1
q_0	q_0, q_3	q_0, q_1
q_0, q_3	q_0, q_2, q_3	q_0, q_1
q_0, q_1	q_0, q_3	q_0, q_1, q_2
q_0, q_2, q_3	q_0, q_2, q_3	q_0, q_1, q_2
q_0, q_1, q_2	q_0, q_2, q_3	q_0, q_1, q_2

	0	1
q_0	q_0, q_3	q_0, q_1
q_0, q_3	q_0, q_2, q_3	q_0, q_1
q_0, q_1	q_0, q_3	q_0, q_1, q_2
q_0, q_2, q_3	q_0, q_2, q_3	q_0, q_1, q_2
q_0, q_1, q_2	q_0, q_2, q_3	q_0, q_1, q_2

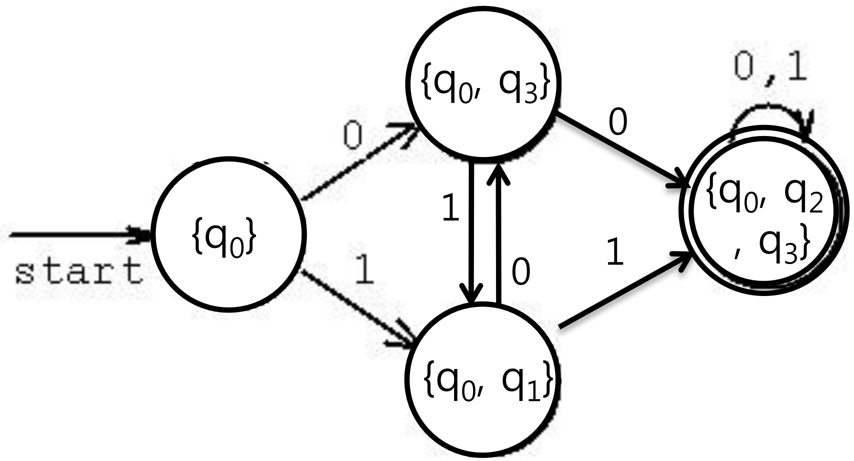
	0	1
$\{q_0\}$	$\{q_0, q_3\}$	$\{q_0, q_1\}$
$\{q_0, q_3\}$	$\{q_0, q_2, q_3\}$	$\{q_0, q_1\}$
$\{q_0, q_1\}$	$\{q_0, q_3\}$	$\{q_0, q_1, q_2\}$
$\{q_0, q_2, q_3\}$	$\{q_0, q_2, q_3\}$	$\{q_0, q_1, q_2\}$
$\{q_0, q_1, q_2\}$	$\{q_0, q_2, q_3\}$	$\{q_0, q_1, q_2\}$



	0	1
$\{q_0\}$	$\{q_0, q_3\}$	$\{q_0, q_1\}$
$\{q_0, q_3\}$	$\{q_0, q_2, q_3\}$	$\{q_0, q_1\}$
$\{q_0, q_1\}$	$\{q_0, q_3\}$	$\{q_0, q_1, q_2\}$
$\{q_0, q_2, q_3\}$	$\{q_0, q_2, q_3\}$	$\{q_0, q_1, q_2\}$
$\{q_0, q_1, q_2\}$	$\{q_0, q_2, q_3\}$	$\{q_0, q_1, q_2\}$

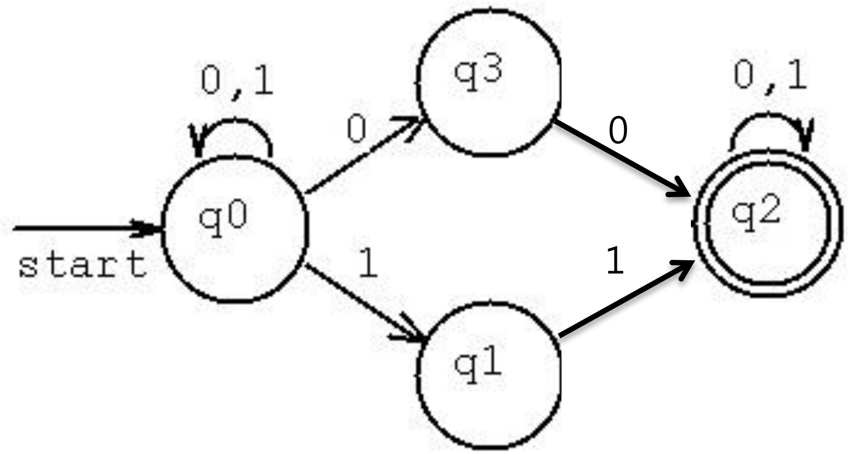


	0	1
$\{q_0\}$	$\{q_0, q_3\}$	$\{q_0, q_1\}$
$\{q_0, q_3\}$	$\{q_0, q_2, q_3\}$	$\{q_0, q_1\}$
$\{q_0, q_1\}$	$\{q_0, q_3\}$	$\{q_0, q_1, q_2\}$
$\{q_0, q_2, q_3\}$	$\{q_0, q_2, q_3\}$	$\{q_0, q_1, q_2\}$
$\{q_0, q_1, q_2\}$	$\{q_0, q_2, q_3\}$	$\{q_0, q_1, q_2\}$

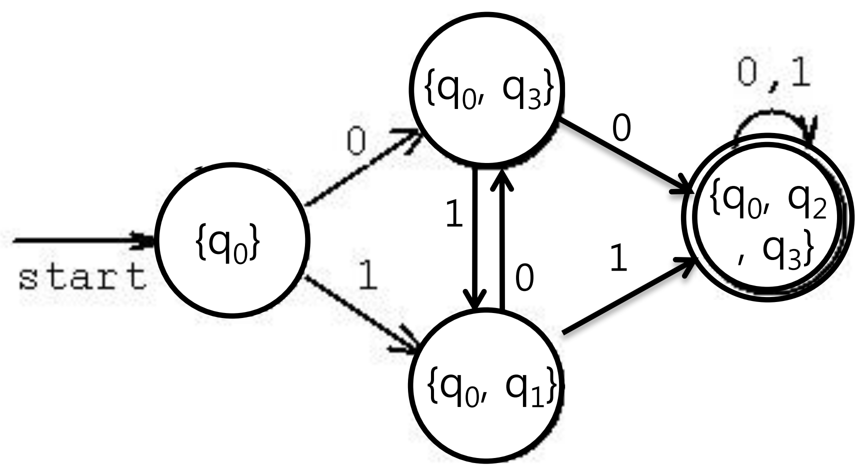


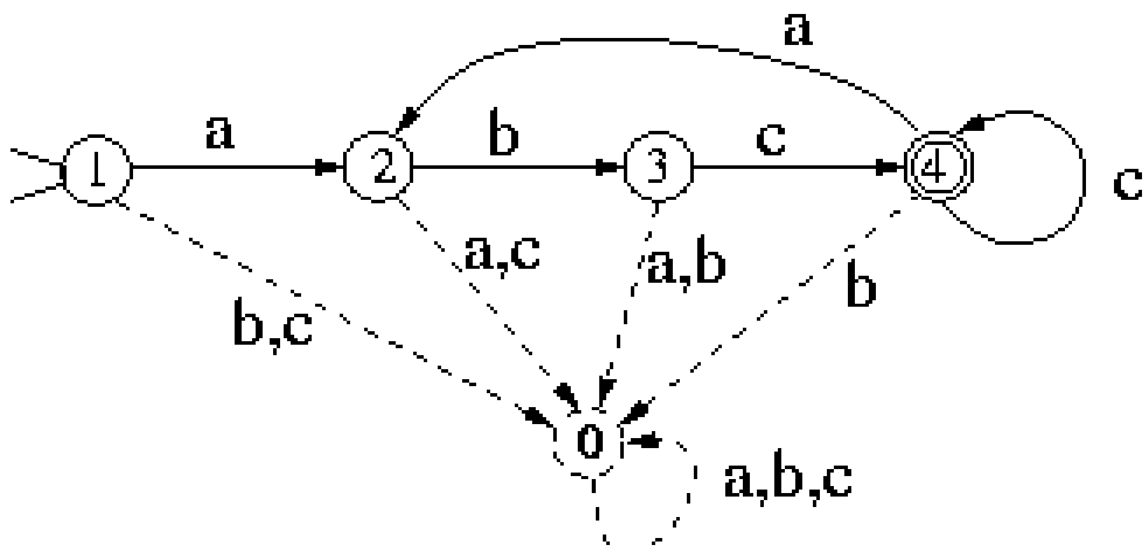
	0	1
$\{q_0\}$	$\{q_0, q_3\}$	$\{q_0, q_1\}$
$\{q_0, q_3\}$	$\{q_0, q_2, q_3\}$	$\{q_0, q_1\}$
$\{q_0, q_1\}$	$\{q_0, q_3\}$	$\{q_0, q_2, q_3\}$
$\{q_0, q_2, q_3\}$	$\{q_0, q_2, q_3\}$	$\{q_0, q_2, q_3\}$

NFA

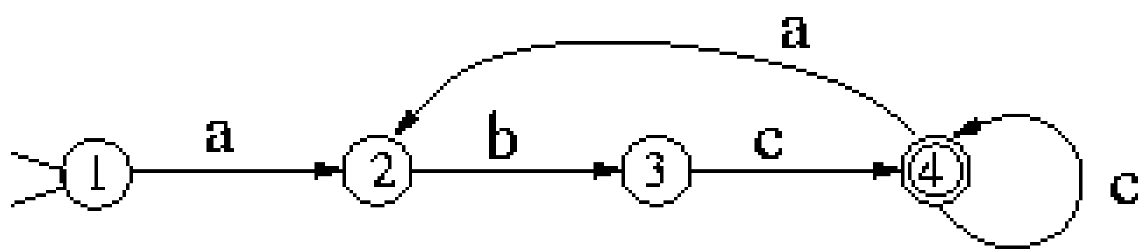

0,1
q3
0
0,1
0
q0
start
q2
1
1
q1

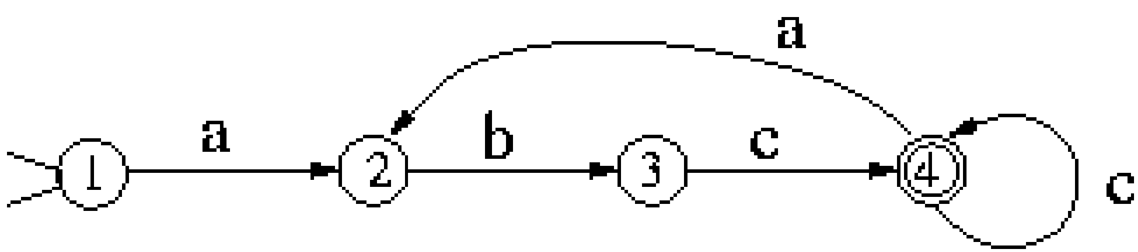
DFA


{q0, q3}
0,1
0
0
{q0}
1
start
{q0, q2, q3}
1
0
1
{q0, q1}

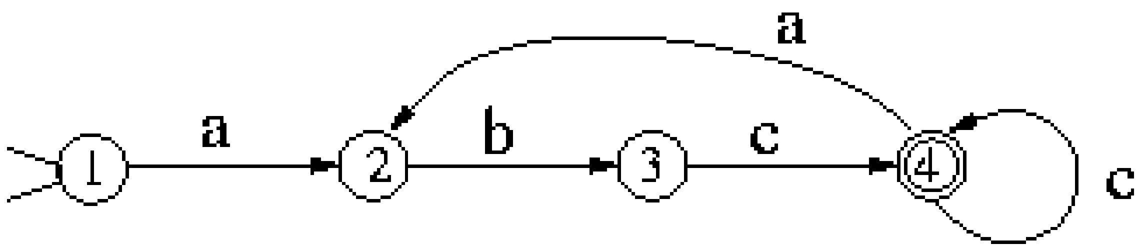


1
2
3
4
0
a
b
c
a
c
b,c
a,c
a,b
b
a,b,c

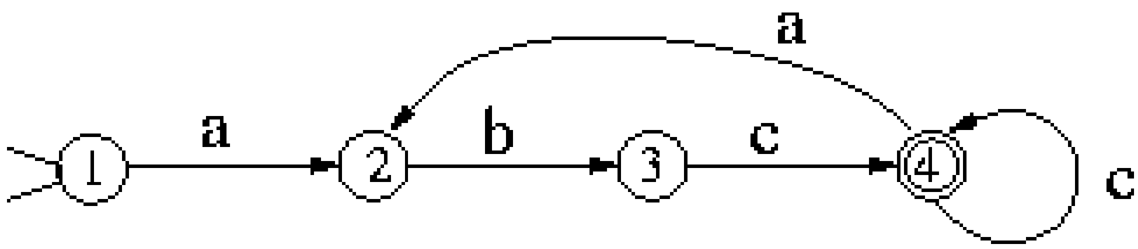

a
a
b
c
c
1
2
3
4



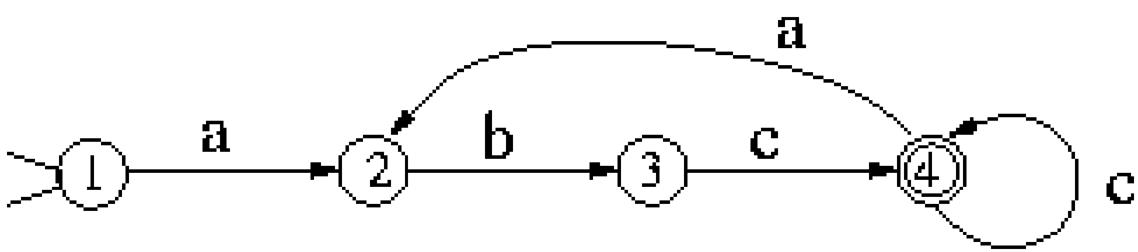
abc



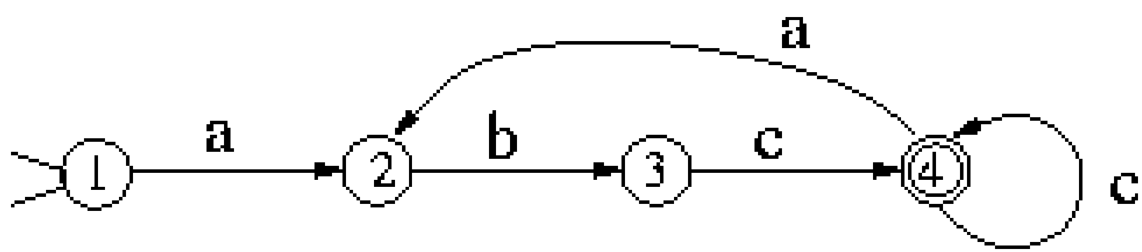
abcccccccccc



abcccccccccccabc



abcc…cabcc…cabccc…cc



$(abc^+)^+$

Regular Expression

$|, \; (), \; ^*, \; ^+, \; \varepsilon$

|, (), *, $^{+}$, ε

|, ( ), *, +, ε

OR

|, ( ), *, $^+$, ε

OR

gray|grey -> {'gray', 'grey'}

|, ( ), *, +, ε

|, ( ), *, +, ε

scope

|, ( ), *, $^+$, ε

scope

gray|grey = gr(a|e)y

|, ( ), *, $^+$, ε

|, (), *, ⁺, ε

zero or more of the preceding element

|, ( ), *, $^+$, ε

zero or more of the preceding element

ab*c →

|, ( ), *, +, ε

zero or more of the preceding element

ab*c → {"*ac*", "*abc*", "*abbc*", "*abbbc*", ...}

|, (), *, $^+$, ε

|, ( ), *, +, ε

one or more of the preceding element

|, ( ), *, +, ε

one or more of the preceding element

$ab^{+}c \rightarrow$

|, ( ), *, $^+$, ε

one or more of the preceding element

ab$^+$c → {"*abc*", "*abbc*", "*abbbc*", ...}, but not "*ac*"

|, (), *, +, ε

one or more of the preceding element

$ab^{+}c = abb^{*}c$

|, (), *, +, ε

one or more of the preceding element

$ab^+c = abb^*c = ab^*bc$

|, ( ), *, $^+$, ε

|, ( ), *, +, ε

empty string

|, ( ), *, $^+$, **ε**

empty string : ""

|, ( ), *, $^+$, ε

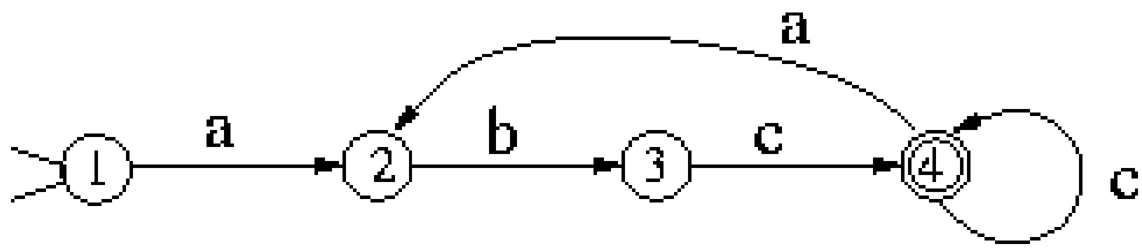
empty string : ""

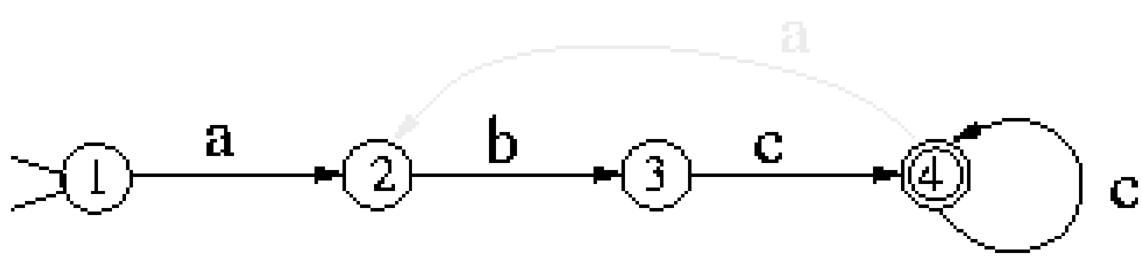
a(b|ε)c →

|, ( ), *, $^+$, **ε**

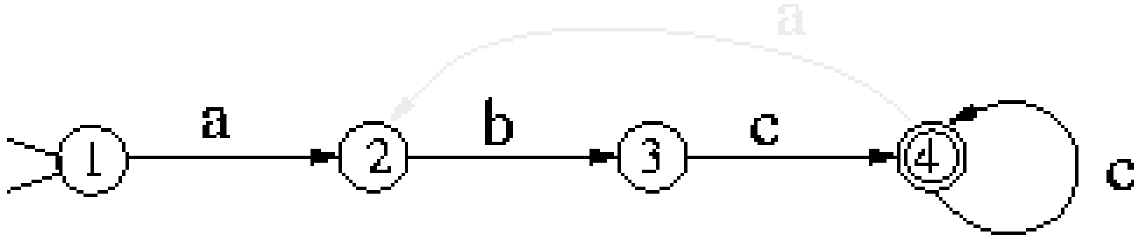
empty string : ""

a(b|ε)c → {"*abc*", "*ac*"}

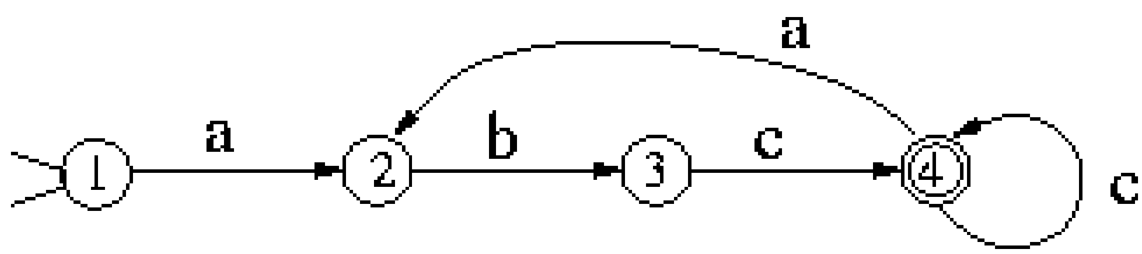

a
b
c
a
c
1
2
3
4

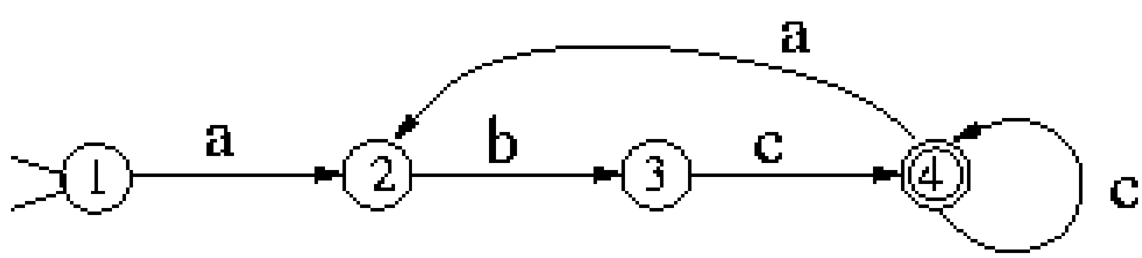


a
a
b
c
c
1
2
3
4

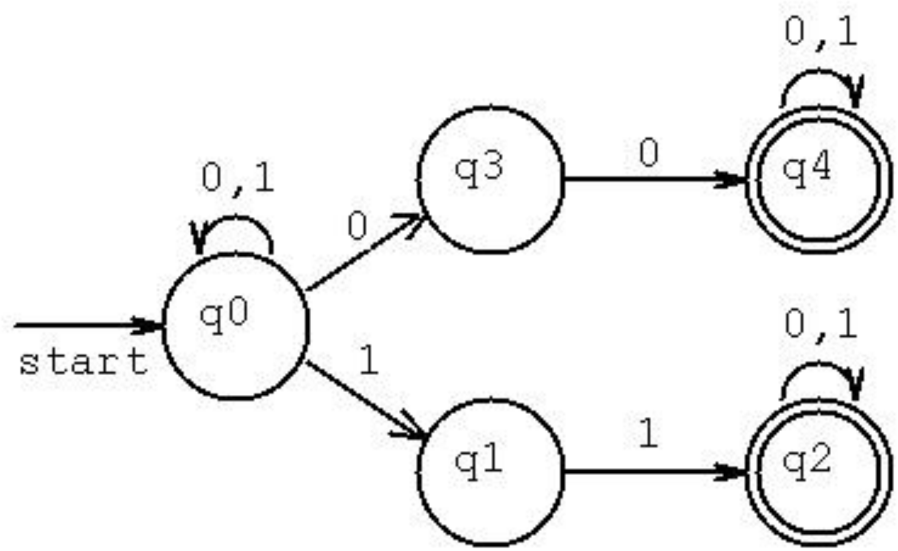


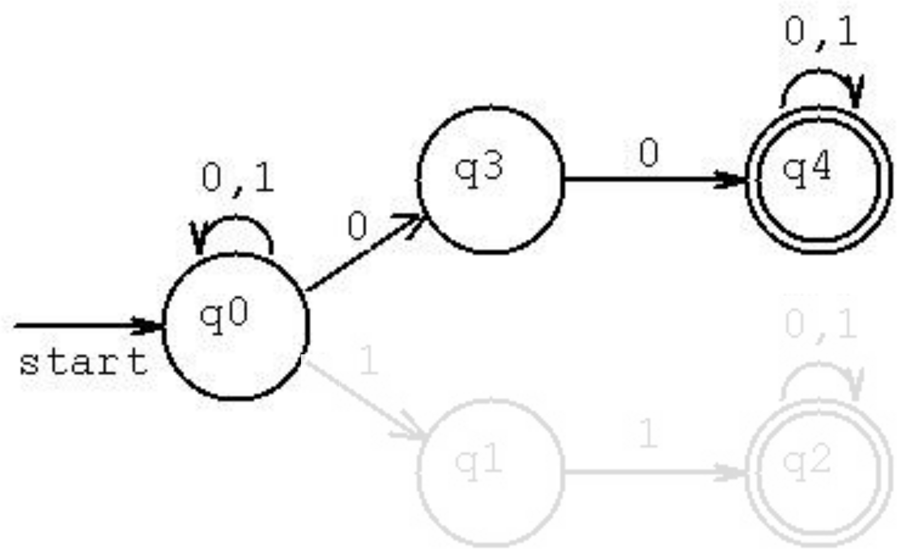
abc$^+$

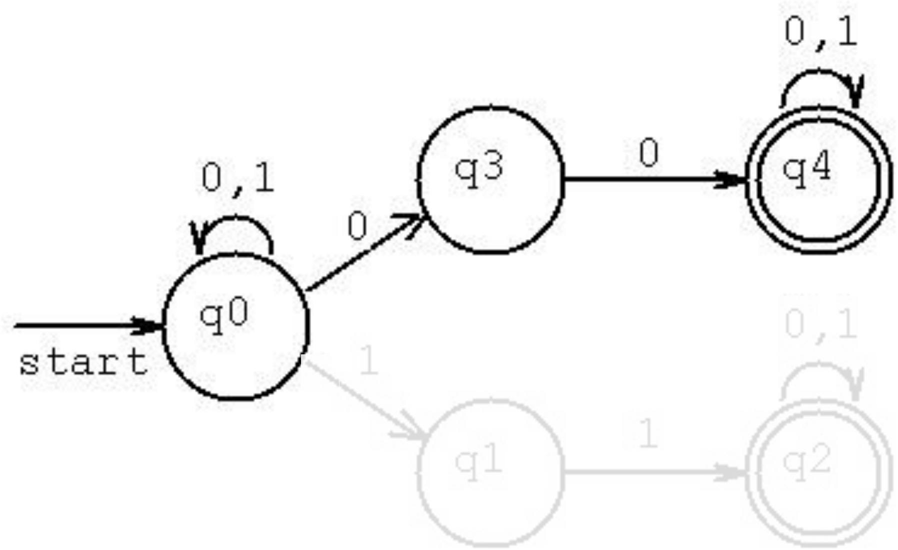

a
a
b
c
c
1
2
3
4



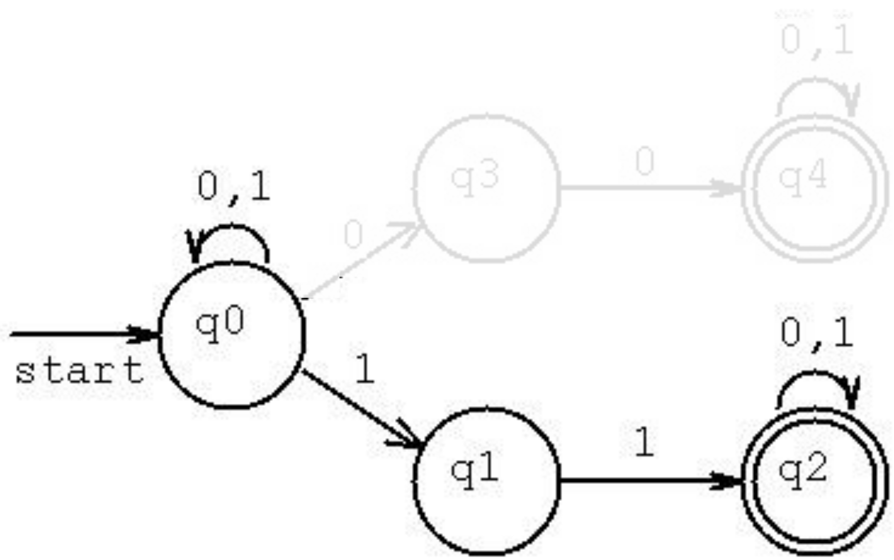
$(abc^{+})^{+}$


0,1
0,1
0,1
q3
0
q4
0
q0
start
1
q1
1
q2

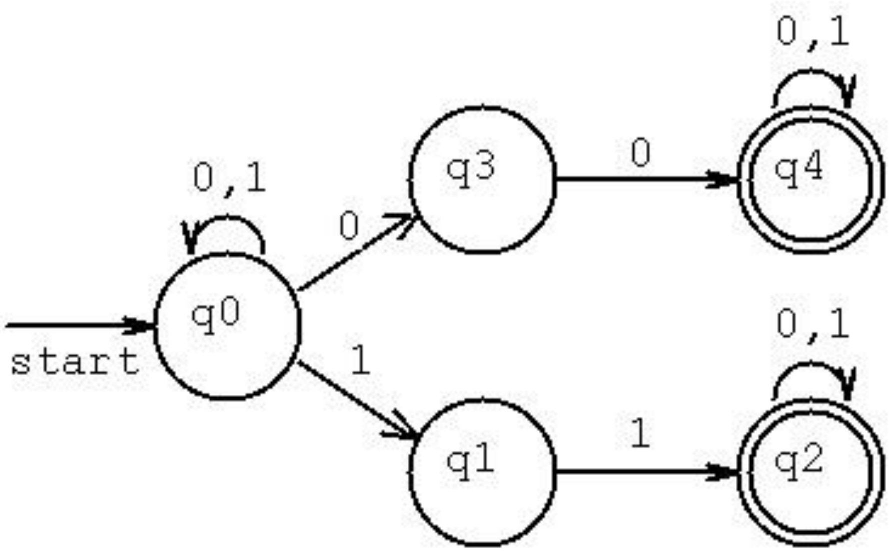

0,1
0,1
0
0
q0
q3
q4
start
1
1
0,1
q1
q2



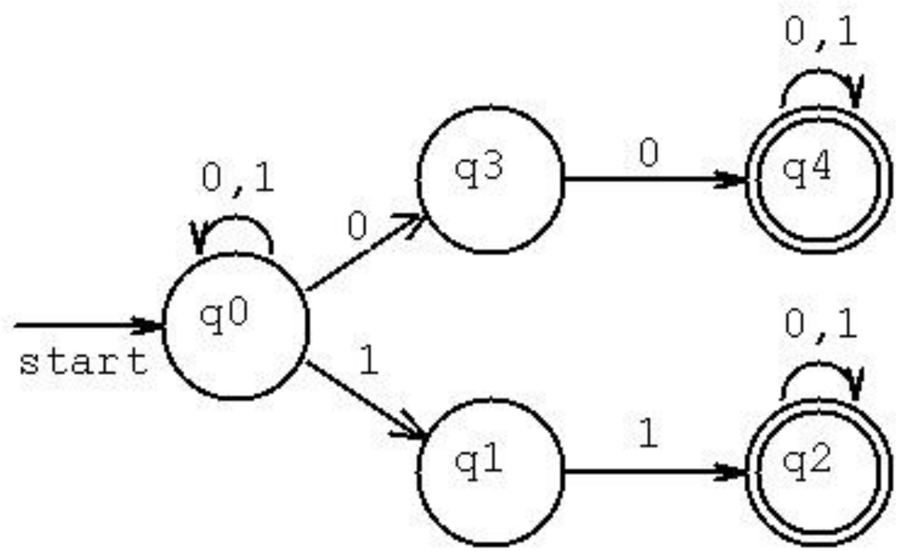
((0|1)*00(0|1)*) | ((0|1)*11(0|1)*)



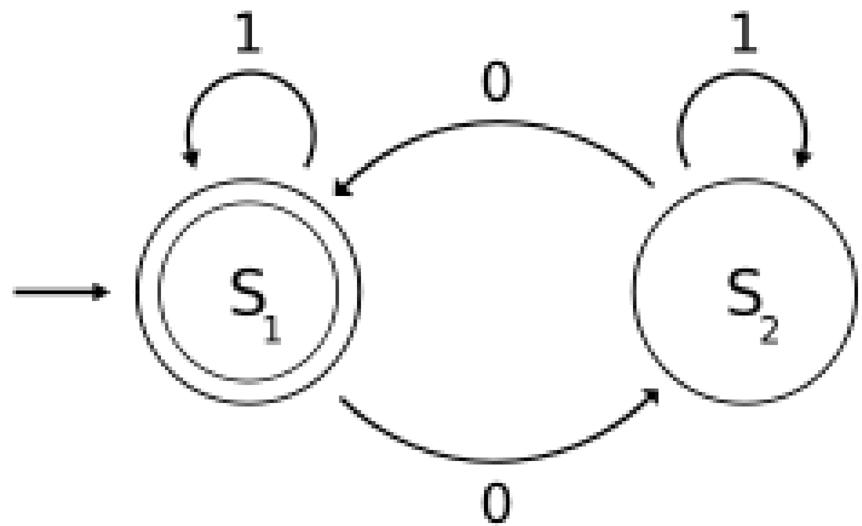
((0|1)*00(0|1)*) | ((0|1)*11(0|1)*)



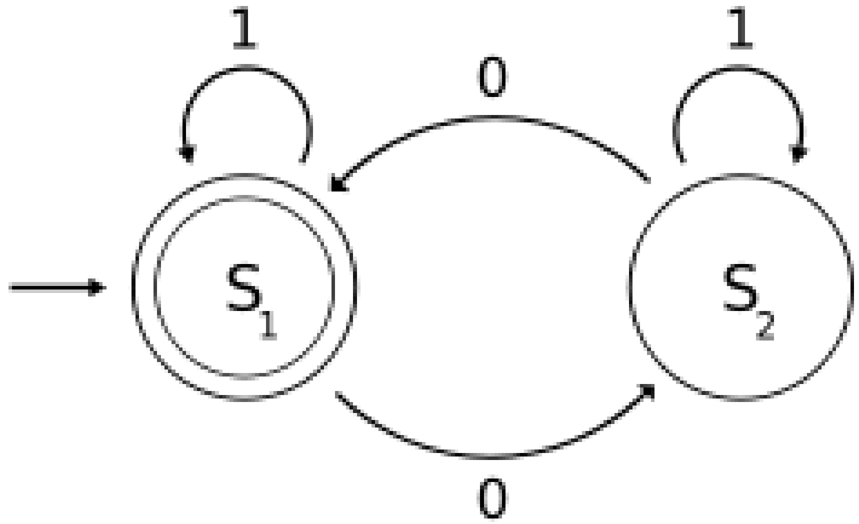
((0|1)*00(0|1)*) | ((0|1)*11(0|1)*)



(0|1)* (00|11) (0|1)*



1
0
1
S_1
S_2
0



(1 | 01*0)*

{01, 0011, 000111, 00001111,...}

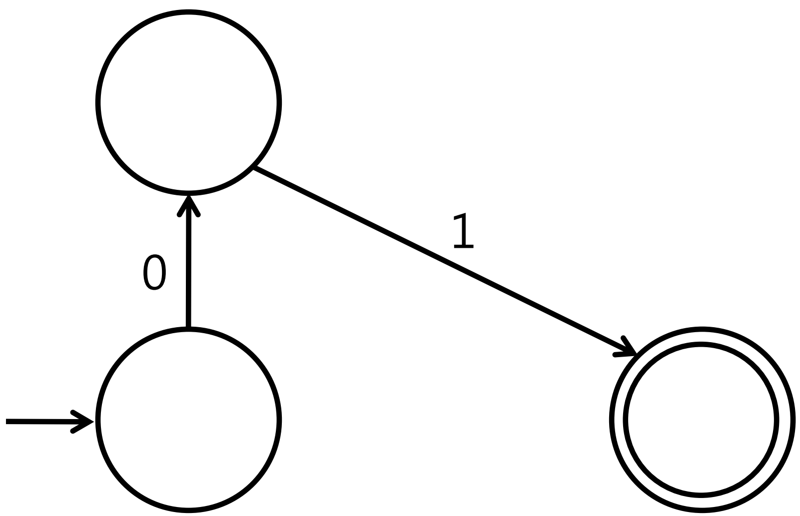
{01, 0011, 000111, 00001111,...}

0^n1^n

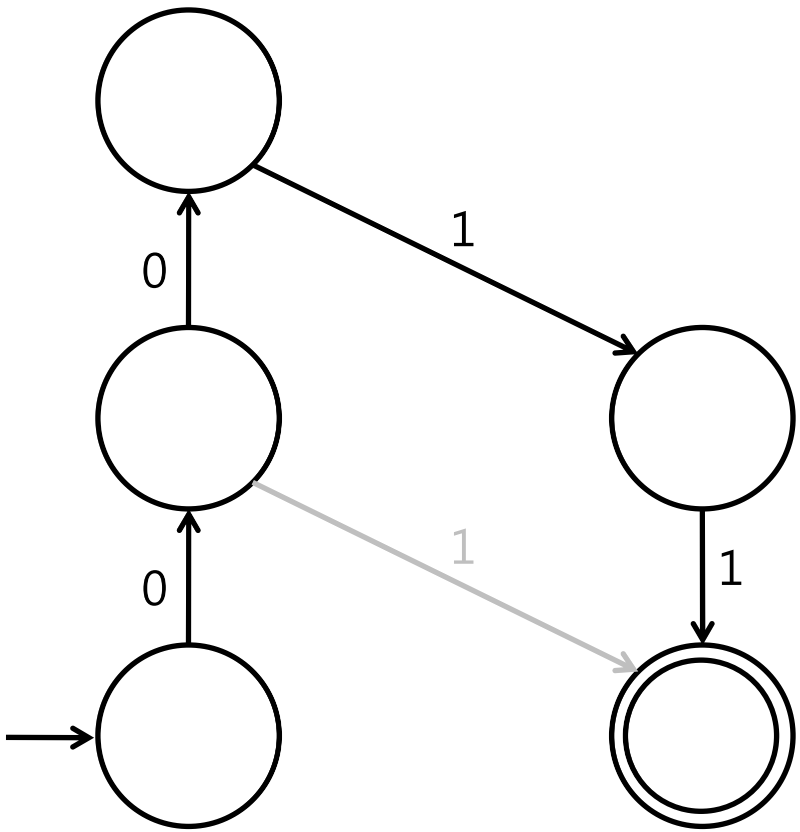
{01, 0011, 000111, 00001111,...}

0^n1^n

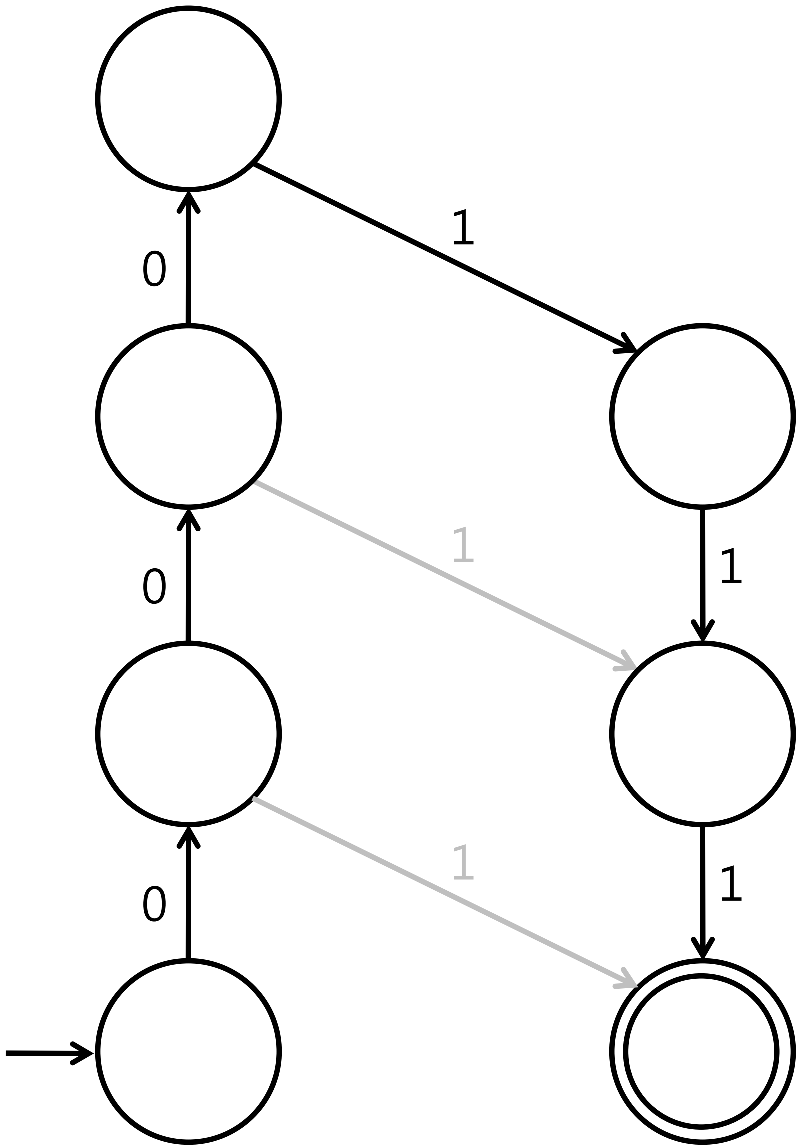
Is it Regular??



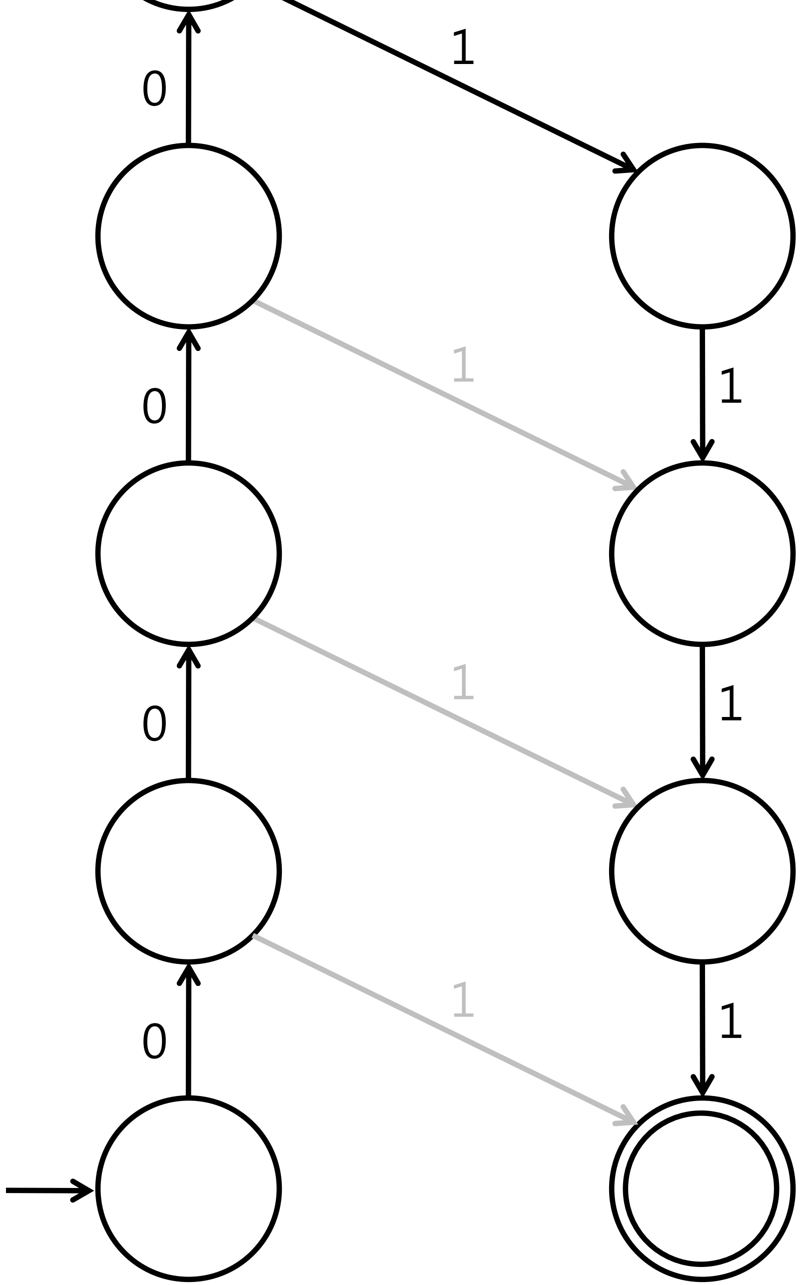
{01}



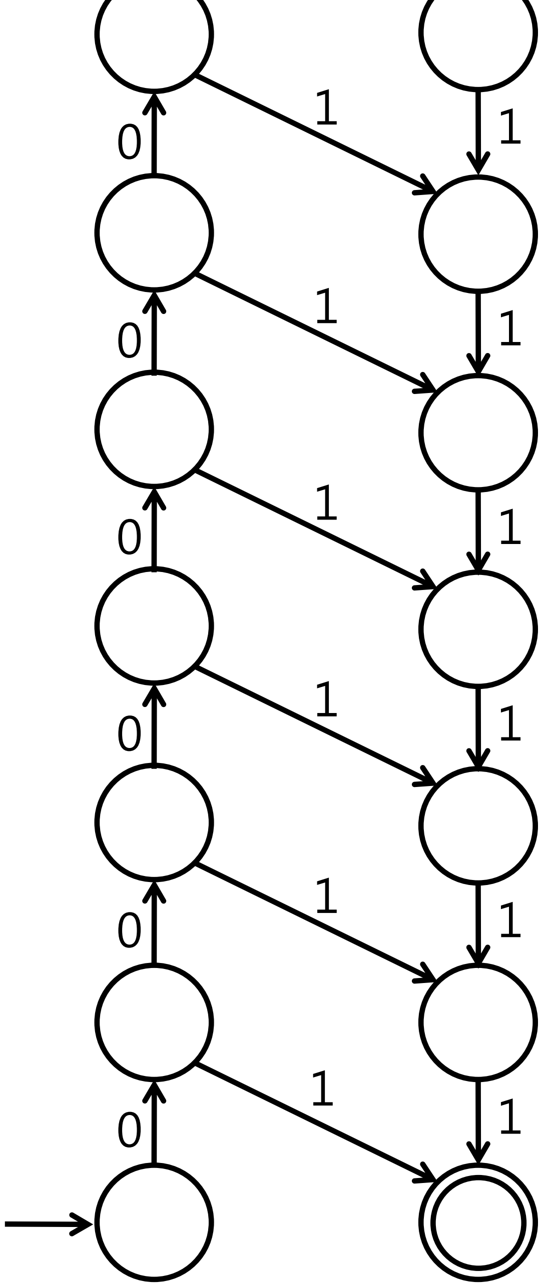
{01, 0011}



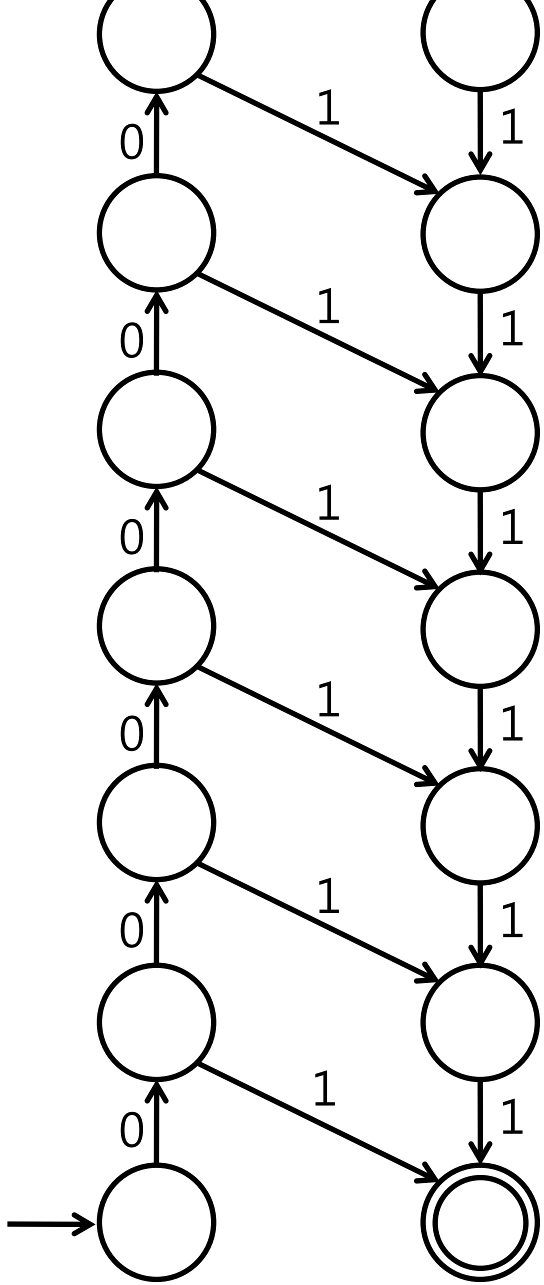
{01, 0011, 000111}



{01, 0011, 000111,
00001111}

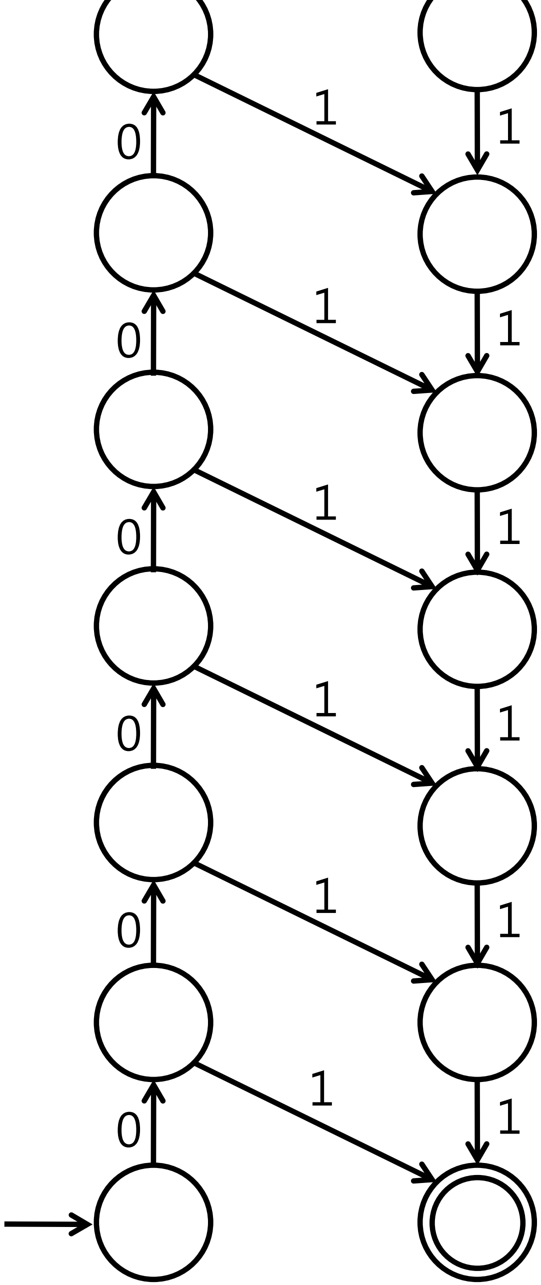


{01, 0011, 000111, 00001111, ...}



cannot be **FINITE** automata

0^n1^n



cannot be **FINITE** automata

NOT regular

0^n1^n

Pumping Lemma

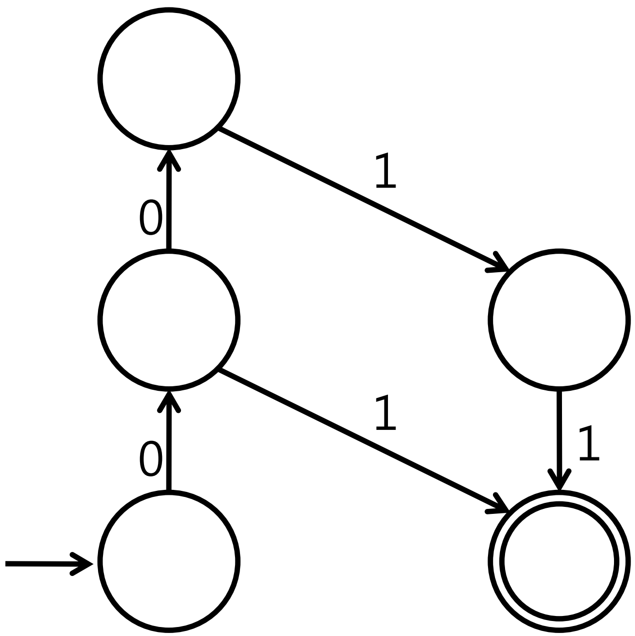
Pumping Lemma

concept

Finite Automata

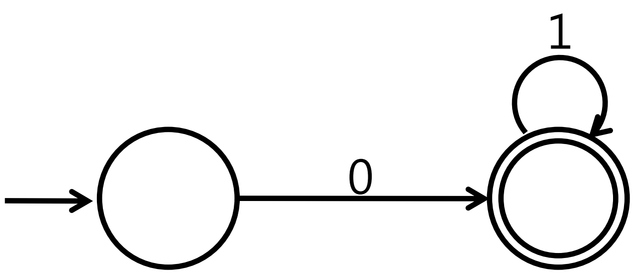
&

Infinite Sentence



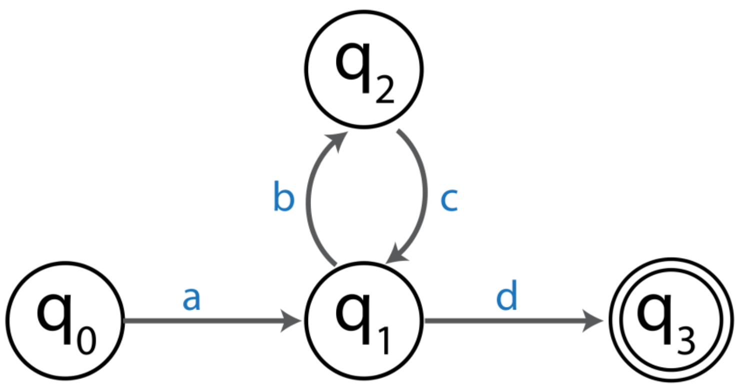
L(01|0011)

={01, 0011}



L(01*)

={01, 011, 0111, 01111, ...}



L(a(bc)*d)

={ad, abcd,
abcbcd, ...}

Pumping Lemma

∃ : exist

∀ : for all

If L is regular

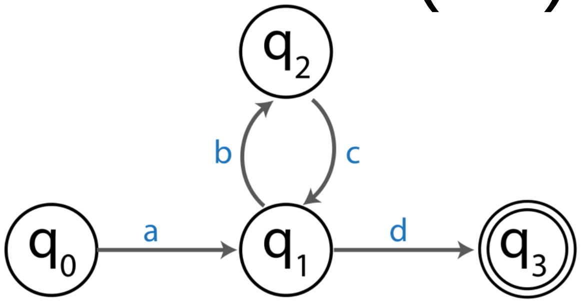
$\exists\, p \geq 1$

s.t. $\forall\, w \in L$, $|w| > p$

$w = xyz$

- $|y| \geq 1$
- $|xy| \leq p$
- $\forall\, i \geq 0$, $xy^{i}z \in L$

a(bc)*d



If L is regular

$\exists\, p \geq 1$

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If L is regular

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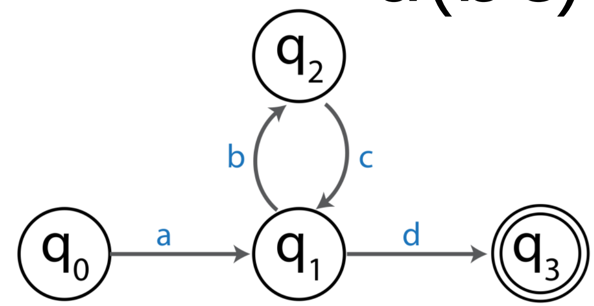
$w = xyz$

- $|y| \geq 1$
- $|xy| \leq p$
- $\forall i \geq 0, xy^iz \in L$

a(bc)*d



If L is regular

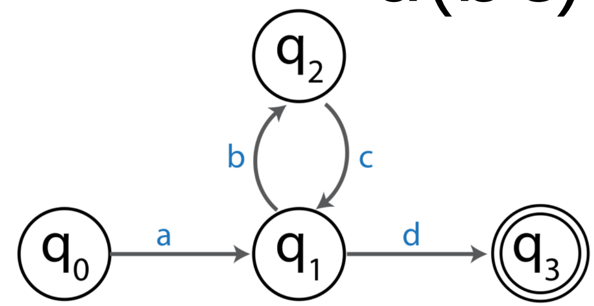
$\exists\, p \geq 1$

s.t. $\forall\, w \in L,\ |w| > p$

$w = xyz$

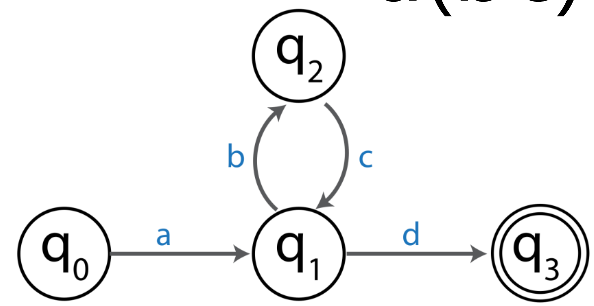
- $|y| \geq 1$
- $|xy| \leq p$
- $\forall\, i \geq 0,\ xy^i z \in L$

a(bc)*d



$p=3$

a(bc)*d



If L is regular

$\exists\, p \geq 1$

s.t. $\forall\, w \in L,\ |w| > p$

$p=3$

$w = xyz$

- $|y| \geq 1$
- $|xy| \leq p$
- $\forall\, i \geq 0,\ xy^{i}z \in L$

If L is regular

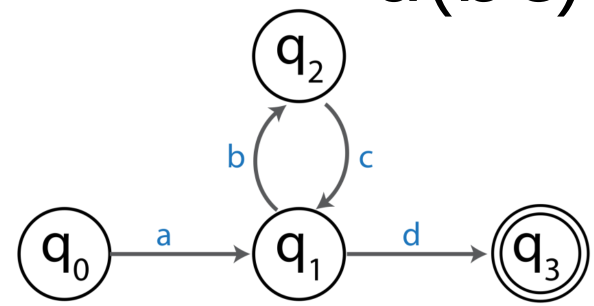
$\exists\, p \geq 1$

s.t. $\forall\, w \in L,\ |w| > p$

$w = xyz$

- $|y| \geq 1$
- $|xy| \leq p$
- $\forall\, i \geq 0,\ xy^i z \in L$

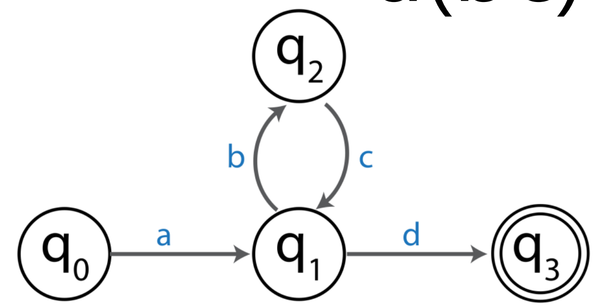
a(bc)*d



$p=3$

$w \in$ {abcd,
abcbcd, ...}

a(bc)*d



If L is regular

$\exists\, p \geq 1$

s.t. $\forall\, w \in L,\ |w| > p$

$p=3$

$w = \{\text{abcd},$

$\text{abcbcd}, \ldots\}$

$w = xyz$

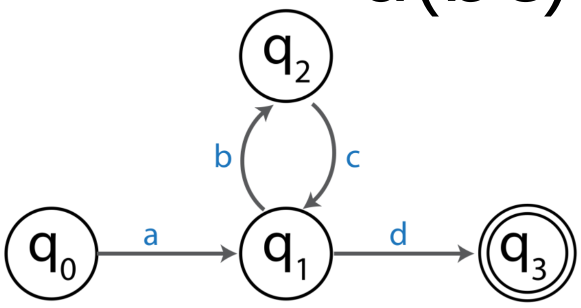
- $|y| \geq 1$
- $|xy| \leq p$
- $\forall\, i \geq 0,\ xy^iz \in L$

If L is regular

$\exists\, p \geq 1$

s.t. $\forall\, w \in L,\ |w| > p$

a(bc)*d



$p=3$

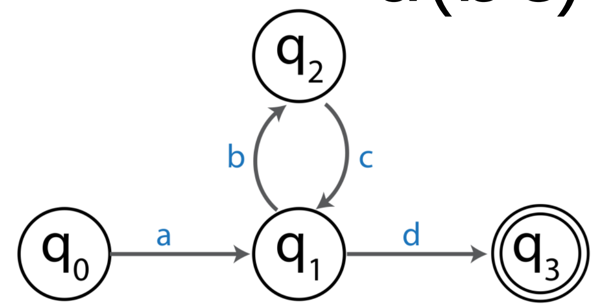
$w = \{abcd,$

$abcbcd, \ldots\}$

$w = xyz$

- $|y| \geq 1$
- $|xy| \leq p$
- $\forall\, i \geq 0,\ xy^iz \in L$

a(bc)*d



If L is regular

$\exists p \geq 1$

s.t. $\forall w \in L, |w| > p$

$p=3$

$w \in \{abcd, abcbcd, \ldots\}$

$w = xyz$

- $|y| \geq 1$
- $|xy| \leq p$
- $\forall i \geq 0, xy^iz \in L$

$x =$ a

$y =$ bc

$z =$ d

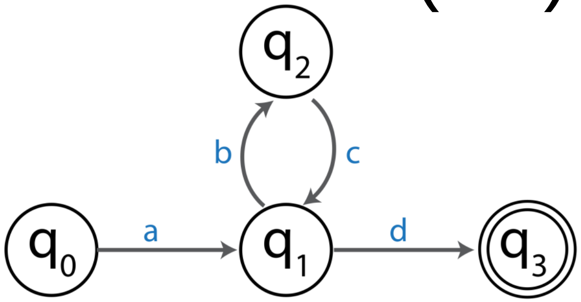
If L is regular

$\exists\, p \geq 1$
s.t. $\forall\, w \in L,\ |w| > p$

$w = xyz$

- $|y| \geq 1$
- $|xy| \leq p$
- $\forall\, i \geq 0,\ xy^i z \in L$

a(bc)*d



$p=3$

$w \in \{$abcd, abcbcd, ...$\}$

x = a

y = bc

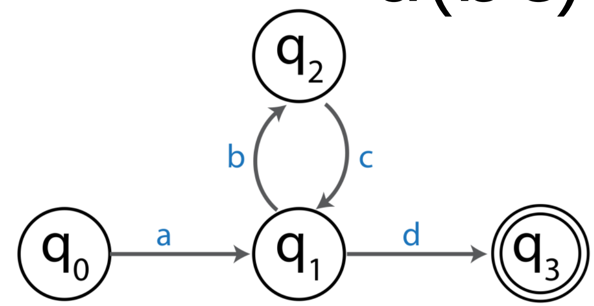
z = d

a(bc)*d

If L is regular

$\exists\, p \geq 1$

s.t. $\forall\, w \in L,\ |w| > p$



$p=3$

$w \in \{abcd,$
$abcbcd, \ldots\}$

$w = xyz$

- $|y| \geq 1$
- $|xy| \leq p$
- $\forall\, i \geq 0,\ xy^iz \in L$

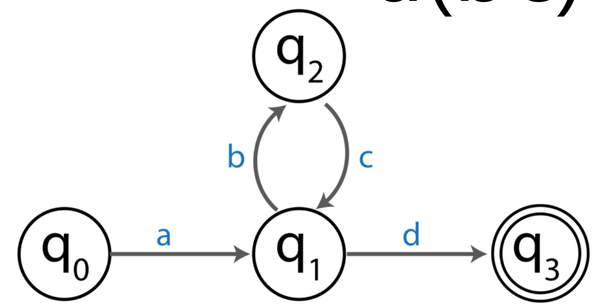
$x = a$

$y = bc$

$z = d$

$|y| = 2$

a(bc)*d



If L is regular

$\exists p \geq 1$
s.t. $\forall w \in L, |w| > p$

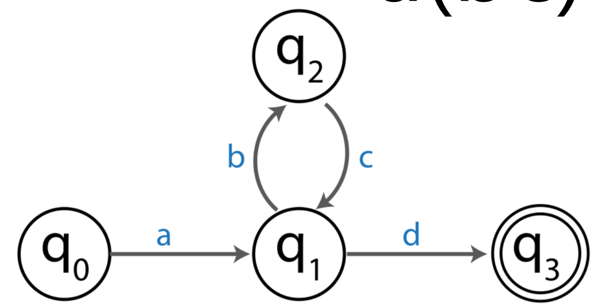
$p=3$
$w \in \{abcd,$
$abcbcd, \ldots\}$

$w = xyz$

- $|y| \geq 1$
- $|xy| \leq p$
- $\forall i \geq 0, xy^iz \in L$

x = a
y = bc
z = d

a(bc)*d



If L is regular

$\exists\, p \geq 1$
s.t. $\forall\, w \in L,\ |w| > p$

$p=3$
$w \in \{\text{abcd},$
$\text{abcbcd}, \ldots\}$

$w = xyz$

- $|y| \geq 1$
- $|xy| \leq p$
- $\forall\, i \geq 0,\ xy^iz \in L$

$x = \text{a}$
$y = \text{bc}$
$z = \text{d}$

$xy = \text{abc}$
$|xy| = 3$

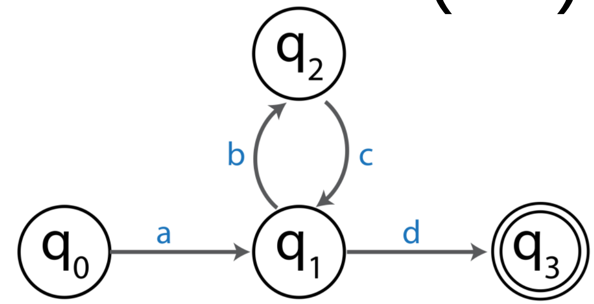
If L is regular

$\exists\, p \geq 1$
s.t. $\forall\, w \in L,\ |w| > p$

$w = xyz$

- $|y| \geq 1$
- $|xy| \leq p$
- $\forall\, i \geq 0,\ xy^iz \in L$

a(bc)*d



$p=3$

$w \in \{$abcd,
abcbcd, ...$\}$

x = a

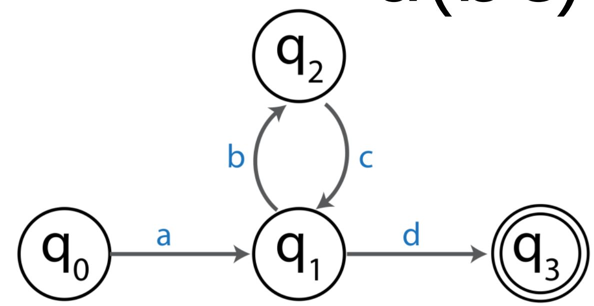
y = bc

z = d

a(bc)*d

If L is regular

$\exists\, p \geq 1$
s.t. $\forall\, w \in L,\ |w| > p$



$p=3$

$w \in \{$abcd,
abcbcd, ...$\}$

$w = xyz$

- $|y| \geq 1$
- $|xy| \leq p$
- $\forall\, i \geq 0,\ xy^iz \in L$

x = a

y = bc

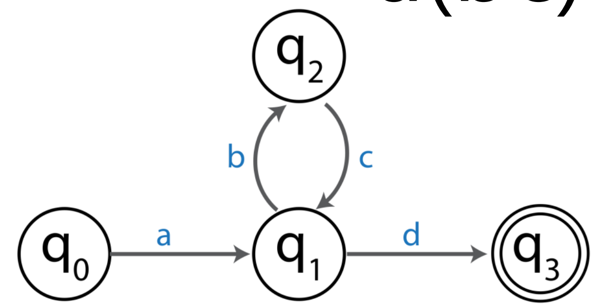
z = d

xy^iz = a(bc)*d

If L is regular

$\exists\, p \geq 1$
s.t. $\forall\, w \in L,\ |w| > p$

a(bc)*d



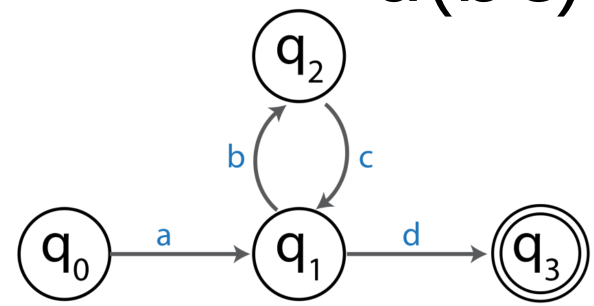
$p=3$

$w = \{$abcd,
abcbcd, ...$\}$

$w = xyz$

- $|y| \geq 1$
- $|xy| \leq p$
- $\forall\, i \geq 0,\ xy^{i}z \in L$

a(bc)*d



If L is regular

$\exists p \geq 1$

s.t. $\forall w \in L, |w| > p$

$p=3$

$w \in \{abcd, abcbcd, \ldots\}$

$w = xyz$

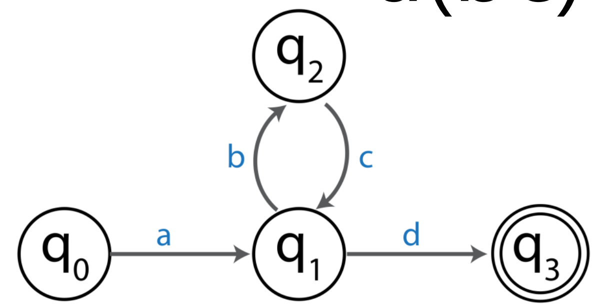
- $|y| \geq 1$
- $|xy| \leq p$
- $\forall i \geq 0, xy^iz \in L$

$x = a$

$y = bc$

$z = bcd$

a(bc)*d



If L is regular

$\exists p \geq 1$
s.t. $\forall w \in L, |w| > p$

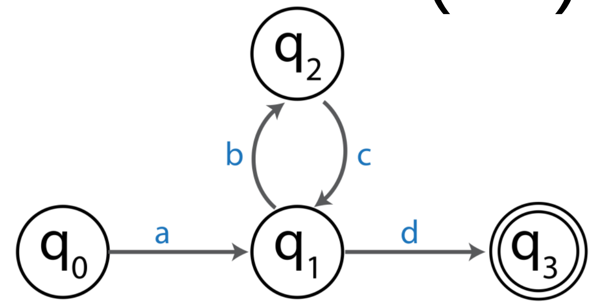
$p=3$
$w \in \{abcd, abcbcd, \ldots\}$

$w = xyz$

- $|y| \geq 1$
- $|xy| \leq p$
- $\forall i \geq 0, xy^iz \in L$

x = a
y = bc
z = bcd

a(bc)*d



If L is regular

$\exists\, p \geq 1$

s.t. $\forall\, w \in L,\ |w| > p$

$p=3$

$w \in \{$abcd,

abcbcd, …$\}$

$w = xyz$

- $|y| \geq 1$
- $|xy| \leq p$
- $\forall\, i \geq 0,\ xy^iz \in L$

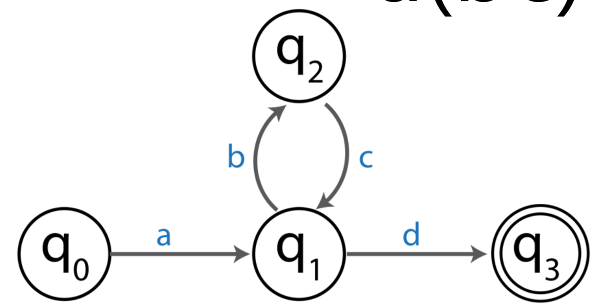
$x =$ a

$y =$ bc

$z =$ bcd

$|y| = 2$

a(bc)*d



If L is regular

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- $|y| \geq 1$
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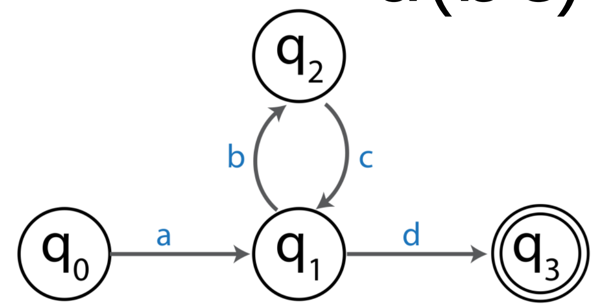
x = a

y = bc

z = bcd

a(bc)*d



If L is regular

$\exists\, p \geq 1$

s.t. $\forall\, w \in L,\ |w| > p$

$p = 3$

$w \in \{abcd,$

$abcbcd, \ldots\}$

$w = xyz$

- $|y| \geq 1$
- $|xy| \leq p$
- $\forall\, i \geq 0,\ xy^iz \in L$

$x = a$

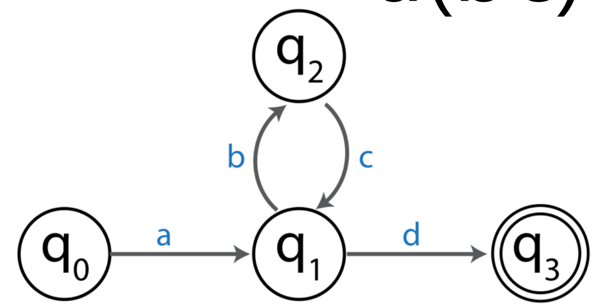
$y = bc$

$z = bcd$

$xy = abc$

$|xy| = 3$

a(bc)*d



If L is regular

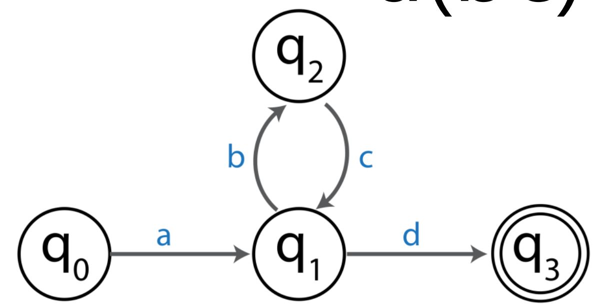
$\exists\, p \geq 1$
s.t. $\forall\, w \in L,\ |w| > p$

$p=3$
$w \in \{$abcd,
abcbcd, ...$\}$

$w = xyz$
- $|y| \geq 1$
- $|xy| \leq p$
- $\forall\, i \geq 0,\ xy^i z \in L$

x = a
y = bc
z = bcd

a(bc)*d



If L is regular

$\exists\, p \geq 1$
s.t. $\forall\, w \in L,\ |w| > p$

$p=3$
$w \in \{$abcd, abcbcd, ...$\}$

$w = xyz$

- $|y| \geq 1$
- $|xy| \leq p$
- $\forall\, i \geq 0,\ xy^iz \in L$

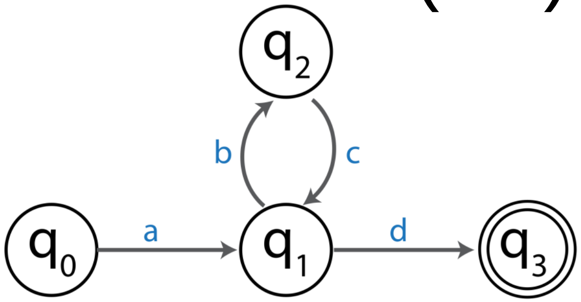
x = a
y = bc
z = bcd

xy^iz = a(bc)$^+$d

a(bc)*d

If L is regular

$\exists\, p \geq 1$
s.t. $\forall\, w \in L,\ |w| > p$



$p=3$

$w \in \{\text{abcd},$
$\text{abcbcd}, \ldots\}$

$w = xyz$

- $|y| \geq 1$
- $|xy| \leq p$
- $\forall\, i \geq 0,\ xy^{i}z \in L$

$x = \text{a}$

$y = \text{bc}$

$z = \text{d}$

Pumping Lemma

Why?

Pumping Lemma

to prove not regular

If L is regular

$\exists\, p \geq 1$

s.t. $\forall\, w \in L,\ |w| > p$

$w = xyz$

- $|y| \geq 1$
- $|xy| \leq p$
- $\forall\, i \geq 0,\ xy^{i}z \in L$

If L is regular

$\exists\, p \geq 1$

s.t. $\forall\, w \in L,\ |w| > p$

$w = xyz$

- $|y| \geq 1$
- $|xy| \leq p$
- $\forall\, i \geq 0,\ xy^{i}z \in L$

L is not regular if

$\exists\, p \geq 1$

s.t. $\forall\, w \in L,\ |w| > p$

$w = xyz$

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L is not regular if

$\forall p \geq 1$

$\exists w \in L, |w| > p$ s.t.

$w = xyz$

- $|y| \geq 1$
- $|xy| \leq p$
- $\forall i \geq 0, xy^i z \in L$

L is not regular if

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$\exists w \in L, |w| > p$ s.t.

$w = xyz$

- $|y| \geq 1$
- $|xy| \leq p$
- $\forall i \geq 0, xy^iz \in L$

L is not regular if

$\forall p \geq 1$

$\exists w \in L, |w| > p$ s.t.

$\forall x, y, z \quad w = xyz$

- $|y| \geq 1$
- $|xy| \leq p$
- $\forall i \geq 0, xy^iz \in L$

L is not regular if

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$\forall x, y, z \quad w = xyz$

- $|y| \geq 1$
- $|xy| \leq p$
- $\forall i \geq 0, xy^i z \in L$

L is not regular if

$\forall p \geq 1$

$\exists w \in L, |w| > p$ s.t.

$\forall x, y, z \quad w = xyz$

- $|y| = 0$ or
- $|xy| \leq p$
- $\forall i \geq 0, xy^i z \in L$

L is not regular if

$\forall p \geq 1$

$\exists w \in L, |w| > p$ s.t.

$\forall x, y, z \quad w = xyz$

- $|y| = 0$ or
- $|xy| \leq p$
- $\forall i \geq 0, xy^i z \in L$

L is not regular if

$\forall p \geq 1$

$\exists w \in L, |w| > p$ s.t.

$\forall x, y, z \quad w = xyz$

- $|y| = 0$ or
- $|xy| > p$ or
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L is not regular if

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- $|y| = 0$ or
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$\exists w \in L, |w| > p$ s.t.

$\forall x, y, z \quad w = xyz$

- $|y| = 0$ or
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$\forall x, y, z \quad w = xyz$

- $|y| = 0$ or
- $|xy| > p$ or
- $\exists i \geq 0, xy^{i}z \notin L$

Pumping Lemma

to prove not regular

0^n1^n

to prove not regular

L is not regular if

$0^n 1^n$

$\forall p \geq 1$

$\exists w \in L, |w| > p$ s.t.

$\forall x, y, z \quad w = xyz$

- $|y| = 0$ or
- $|xy| > p$ or
- $\exists i \geq 0, xy^iz \notin L$

L is not regular if

$\forall p \geq 1$

$\exists w \in L, |w| > p$ s.t.

$\forall x, y, z \quad w = xyz$

- $|y| = 0$ or
- $|xy| > p$ or
- $\exists i \geq 0, xy^i z \notin L$

$0^n 1^n$

L is not regular if

$\forall p \geq 1$

$\exists w \in L, |w| > p$ s.t.

$\forall x, y, z \quad w = xyz$

- $|y| = 0$ or
- $|xy| > p$ or
- $\exists i \geq 0, xy^i z \notin L$

$0^n 1^n$

assume p

L is not regular if

0^n1^n

$\forall p \geq 1$

assume p

$\exists\, w \in L, |w| > p$ s.t.

$\forall\, x, y, z \quad w = xyz$

- $|y| = 0$ or
- $|xy| > p$ or
- $\exists\, i \geq 0, xy^iz \notin L$

L is not regular if

$\forall p \geq 1$

$\exists w \in L, |w| > p$ s.t.

$\forall x, y, z \quad w = xyz$

- $|y| = 0$ or
- $|xy| > p$ or
- $\exists i \geq 0, xy^i z \notin L$

$0^n 1^n$

assume p

let $w = 0^p 1^p$

L is not regular if

0^n1^n

$\forall p \geq 1$

$\exists w \in L, |w| > p$ s.t.

assume p

let $w = 0^p1^p$

$\forall x, y, z \quad w = xyz$

- $|y| = 0$ or
- $|xy| > p$ or
- $\exists i \geq 0, xy^iz \notin L$

L is not regular if

0^n1^n

$\forall p \geq 1$

$\exists w \in L, |w| > p$ s.t.

assume p

let $w = 0^p1^p$

$\forall x, y, z \quad w = xyz$

- $|y| = 0$ or
- $|xy| > p$ or
- $\exists i \geq 0, xy^iz \notin L$

i)

ii)

L is not regular if

$\forall p \geq 1$

$\exists w \in L, |w| > p$ s.t.

$\forall x, y, z \quad w = xyz$

- $|y| = 0$ or
- $|xy| > p$ or
- $\exists i \geq 0, xy^i z \notin L$

$0^n 1^n$

assume p

let $w = 0^p 1^p$

i) $x = 0^p$

$y = \varepsilon$

$z = 0^{p-m-n} 1^p$

ii)

L is not regular if

$\forall p \geq 1$

$\exists w \in L, |w| > p$ s.t.

$\forall x, y, z \quad w = xyz$

- $|y| = 0$ or
- $|xy| > p$ or
- $\exists i \geq 0, xy^i z \notin L$

$0^n 1^n$

assume p

let $w = 0^p 1^p$

i) $x = 0^p$

$y = \varepsilon$

$z = 0^{p-m-n} 1^p$

ii)

L is not regular if

$0^n 1^n$

$\forall p \geq 1$

$\exists w \in L, |w| > p$ s.t.

assume p

let $w = 0^p 1^p$

$\forall x, y, z \quad w = xyz$

- $|y| = 0$ or
- $|xy| > p$ or
- $\exists i \geq 0, xy^i z \notin L$

i) $x = 0^m$

$y = 0^n$ $(n > 0)$

$z = 0^{p-m-n} 1^p$

ii)

L is not regular if

$\forall p \geq 1$

$\exists w \in L, |w| > p$ s.t.

$\forall x, y, z \quad w = xyz$

- $|y| = 0$ or
- $|xy| > p$ or
- $\exists i \geq 0, xy^iz \notin L$

0^n1^n

assume p

let $w = 0^p1^p$

i) $x = 0^m$

$y = 0^n$ $(n>0)$

$z = 0^{p-m-n}1^p$

ii)

L is not regular if

$\forall p \geq 1$

$\exists w \in L, |w| > p$ s.t.

$\forall x, y, z \quad w = xyz$

- $|y| = 0$ or
- $|xy| > p$ or
- $\exists i \geq 0, xy^i z \notin L$

$0^n 1^n$

assume p

let $w = 0^p 1^p$

i) $x = 0^m$

$y = 0^n \ (n > 0)$

$z = 0^{p-m-n} 1^p$

ii)

L is not regular if

$\forall p \geq 1$

$\exists w \in L, |w| > p$ s.t.

$\forall x, y, z \quad w = xyz$

- $|y| = 0$ or
- $|xy| > p$ or
- $\exists i \geq 0, xy^iz \notin L$

0^n1^n

assume p

let $w = 0^p1^p$

i) $x = 0^m$

$y = 0^n$ (n>0)

$z = 0^{p-m-n}1^p$

ii)

$i = 0,\ xy^iz =$

L is not regular if

$\forall p \geq 1$

$\exists w \in L, |w| > p$ s.t.

$\forall x, y, z \quad w = xyz$

- $|y| = 0$ or
- $|xy| > p$ or
- $\exists i \geq 0, xy^i z \notin L$

0^n1^n

assume p

let $w = 0^p1^p$

i) $x = 0^m$

$y = 0^n \ (n>0)$

$z = 0^{p-m-n}1^p$

ii)

$i = 0, \ xy^i z = 0^{p-n}1^p$

L is not regular if

$\forall p \geq 1$

$\exists w \in L, |w| > p$ s.t.

$\forall x, y, z \quad w = xyz$

- $|y| = 0$ or
- $|xy| > p$ or
- $\exists i \geq 0, xy^iz \notin L$

0^n1^n

assume p

let $w = 0^p1^p$

i) $x = 0^m$

$y = 0^n \ (n > 0)$

$z = 0^{p-m-n}1^p$

ii)

$$i = 0, \ xy^iz = 0^{p-n}1^p \notin L(0^n1^n)$$

L is not regular if

$\forall p \geq 1$

$\exists w \in L, |w| > p$ s.t.

$\forall x, y, z \quad w = xyz$

- $|y| = 0$ or
- $|xy| > p$ or
- $\exists i \geq 0, xy^iz \notin L$

0^n1^n

assume p

let $w = 0^p1^p$

i)

ii) $x = 0^p$

$y = 1^n \ (n > 0)$

$z = 1^{p-n}$

L is not regular if

$\forall p \geq 1$

$\exists w \in L, |w| > p$ s.t.

$\forall x, y, z \quad w = xyz$

- $|y| = 0$ or
- $|xy| > p$ or
- $\exists i \geq 0, xy^i z \notin L$

$0^n 1^n$

assume p

let $w = 0^p 1^p$

i)

ii) ?

L is not regular if

$\forall p \geq 1$

$\exists w \in L, |w| > p$ s.t.

$\forall x, y, z \quad w = xyz$

- $|y| = 0$ or
- $|xy| > p$ or
- $\exists i \geq 0, xy^i z \notin L$

$0^n 1^n$

assume p

let $w = 0^p 1^p$

i) $x = 0^p$

$y = 0^n$ (n>0)

$z = 0^{p-m-n} 1^n$

L is not regular if

$\forall p \geq 1$

$\exists w \in L, |w| > p$ s.t.

$\forall x, y, z \quad w = xyz$

- $|y| = 0$ or
- $|xy| > p$ or
- $\exists i \geq 0, xy^iz \notin L$

0^n1^n

assume p

let $w = 0^p1^p$

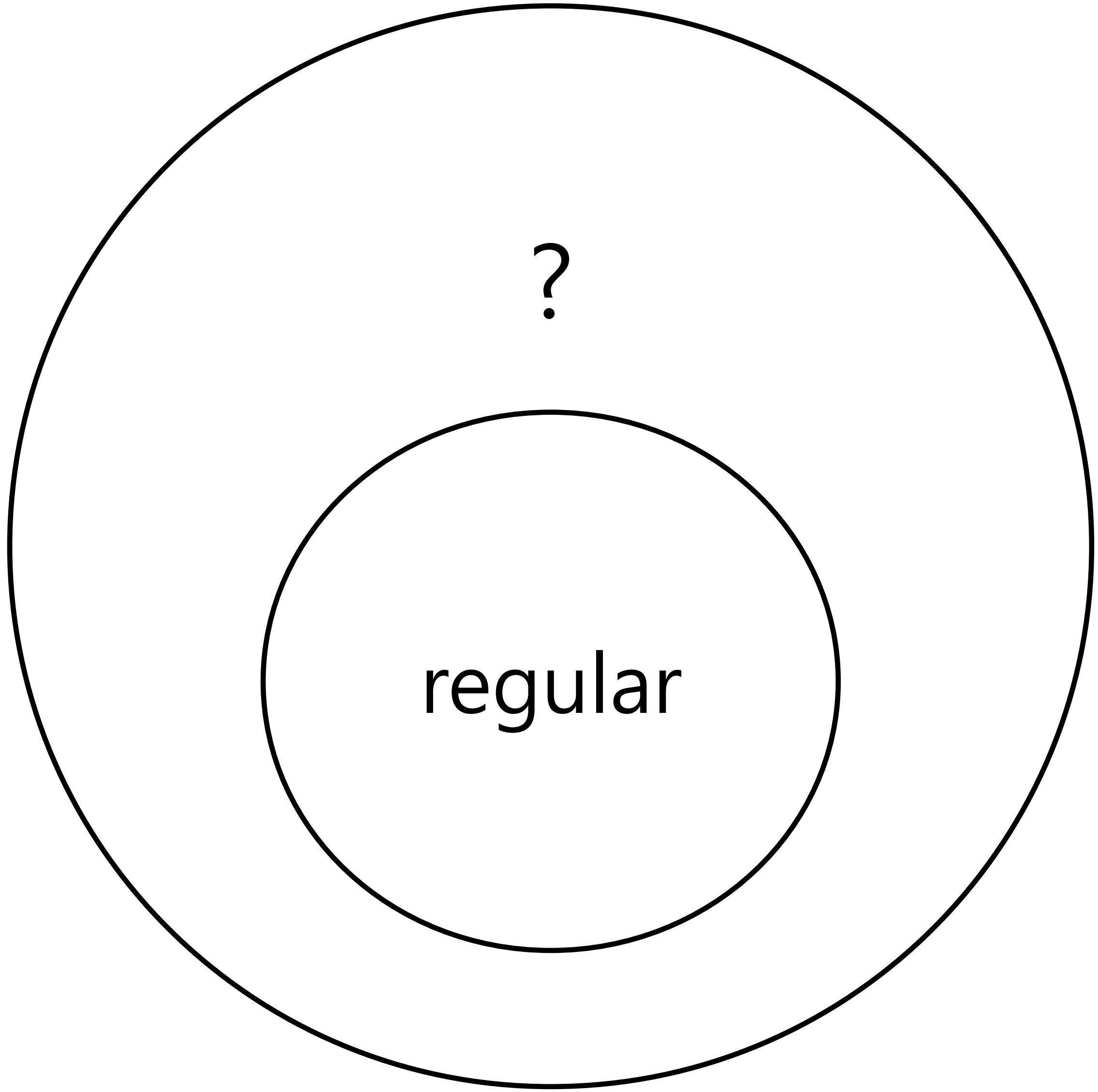
i) $x = 0^p$

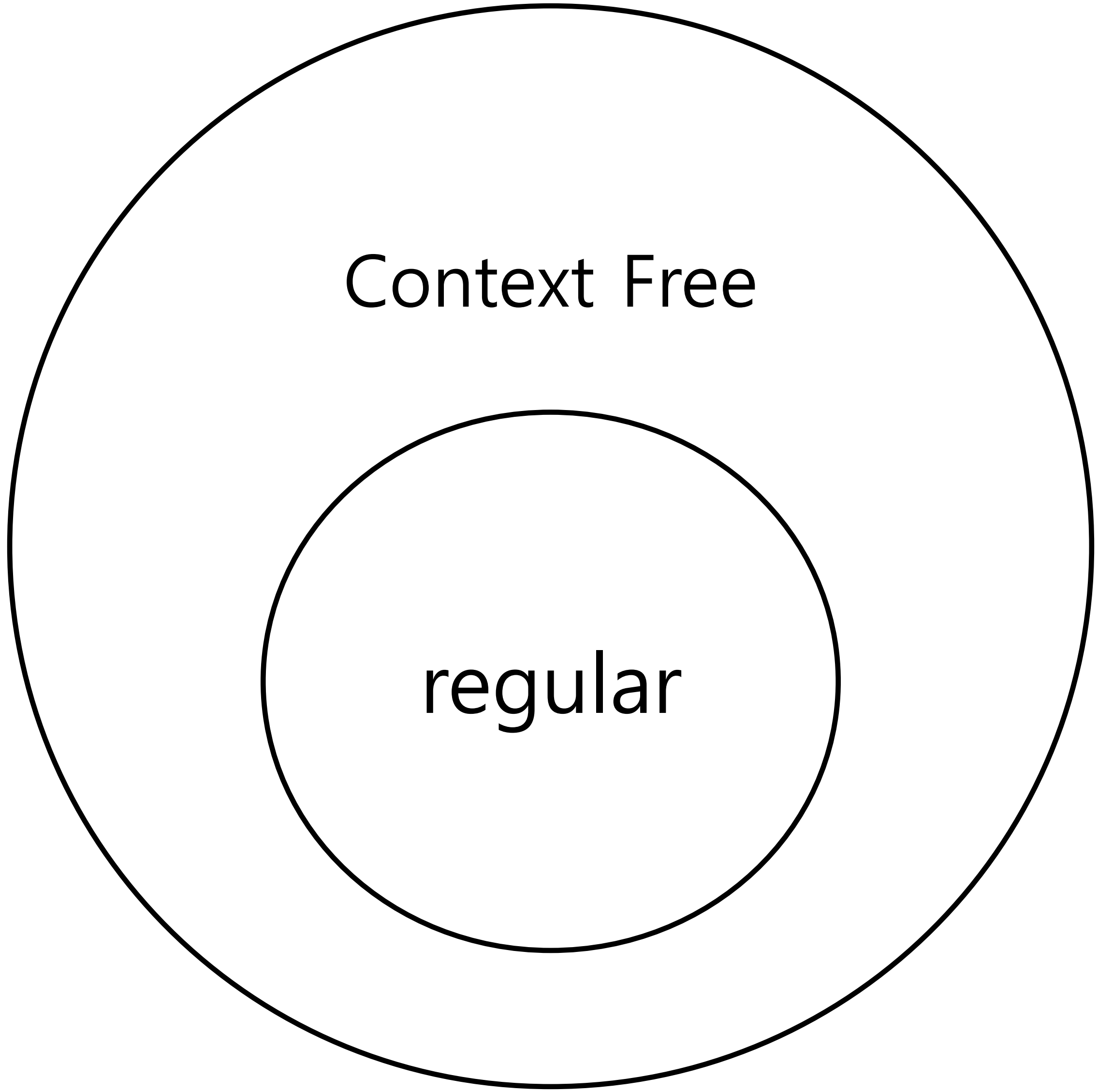
$y = 0^n$ (n>0)

$z = 0^{p-m-n}1^n$

ii) *other wise*

$|y| = 0$ or $|xy| > p$


?
regular


Context Free
regular

Context Free Grammar

$$S \rightarrow 0S1$$
$$S \rightarrow \varepsilon$$

$$S \rightarrow 0S1$$

$$S \rightarrow \varepsilon$$

$$L(S) = 0^n 1^n$$

$$S \rightarrow 0S1 \mid \varepsilon$$

$$L(S) = 0^n1^n$$

$$S \rightarrow S + S$$

$$S \rightarrow S - S$$

$$S \rightarrow S * S$$

$$S \rightarrow S / S$$

$$S \rightarrow (S)$$

$S \rightarrow x \mid y \mid z$

$S \rightarrow S + S$

$S \rightarrow S - S$

$S \rightarrow S * S$

$S \rightarrow S / S$

$S \rightarrow (S)$

$S \to A$

$S \to S + S$

$S \to S - S$

$S \to S * S$

$S \to S / S$

$S \to (S)$

$A \to 0|1|2|3|4|5|6|7|8|9$

S → A

S → S + S

S → S - S

S → S * S

S → S / S

S → (S)

A → 0|1|2|3|4|5|6|7|8|9|AA

$S \rightarrow B$

$S \rightarrow S + S$

$S \rightarrow S - S$

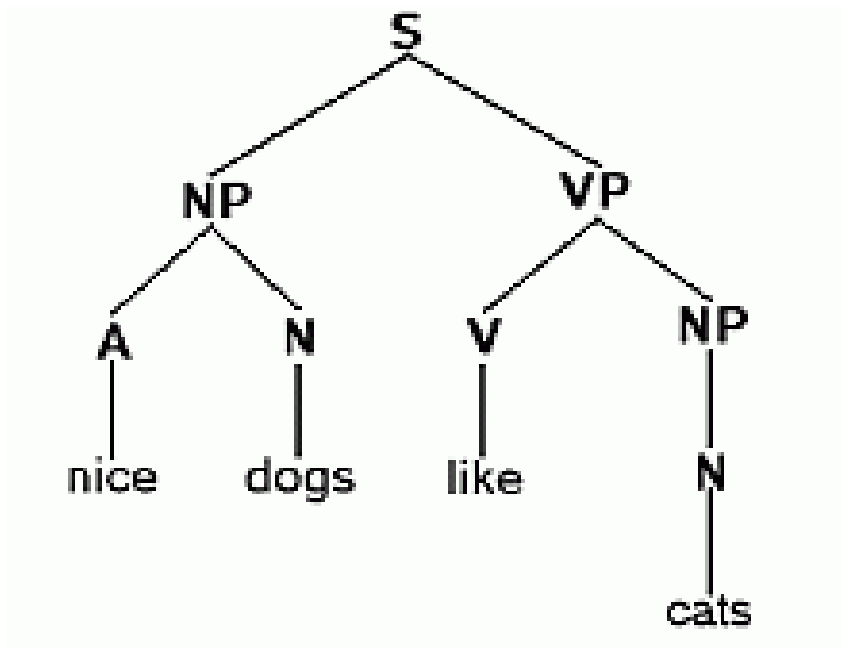
$S \rightarrow S * S$

$S \rightarrow S / S$

$S \rightarrow (S)$

$A \rightarrow 0|1|2|3|4|5|6|7|8|9|AA|\varepsilon$

$B \rightarrow 0 \mid (1|2|3|4|5|6|7|8|9)A$


S
NP
VP
A
N
V
NP
nice
dogs
like
N
cats

$S \rightarrow NP\ VP$

$NP \rightarrow A\ N \mid N$

$VP \rightarrow V\ NP$

$N \rightarrow$ dogs | cats

$V \rightarrow$ like

$A \rightarrow$ nice

(V, Σ, R, S)

$$(V, \Sigma, R, S)$$

(V, Σ, R, S)

a finite set of *non-terminal characters*

(V, Σ, R, S)

a finite set of *variables*

$S \rightarrow NP\ VP$
$NP \rightarrow A\ N \mid N$
$VP \rightarrow V\ NP$
$N \rightarrow$ dogs | cats
$V \rightarrow$ like
$A \rightarrow$ nice

$$(V, \Sigma, R, S)$$

a finite set of *variables*

S → NP VP
NP → A N | N
VP → V NP
N → dogs | cats
V → like
A → nice

(V, Σ, R, S)

a finite set of *variables*

{S, NP, VP, N, V, A}

(V, Σ, R, S)

$$(V, \Sigma, R, S)$$

a finite set of *terminals*

S → NP VP
NP → A N | N
VP → V NP
N → dogs | cats
V → like
A → nice

$$(V, \Sigma, R, S)$$

a finite set of *terminals*

S → NP VP
NP → A N | N
VP → V NP
N → dogs | cats
V → like
A → nice

(V, Σ, R, S)

a finite set of *terminals*

{dogs, cats, like, nice}

(V, Σ, R, S)

(V, Σ, R, S)

a finite relation from V to $(V \cup \Sigma)^*$

(V, Σ, R, S)

a finite rewrite rules(productions)

S → NP VP
NP → A N | N
VP → V NP
N → dogs | cats
V → like
A → nice

(V, Σ, R, S)

a finite rewrite rules(productions)

S → NP VP
NP → A N | N
VP → V NP
N → dogs | cats
V → like
A → nice

(V, Σ, R, S)

a finite rewrite rules(productions)

S → NP VP
NP → A N | N
...
A → nice

(V, Σ, R, S)

(V, Σ, R, S)

a start variable

S → NP VP
NP → A N | N
VP → V NP
N → dogs | cats
V → like
A → nice

$$(V, \Sigma, R, S)$$

a start variable

$S \rightarrow NP\ VP$
$NP \rightarrow A\ N \mid N$
$VP \rightarrow V\ NP$
$N \rightarrow$ dogs | cats
$V \rightarrow$ like
$A \rightarrow$ nice

$$(V, \Sigma, R, S)$$

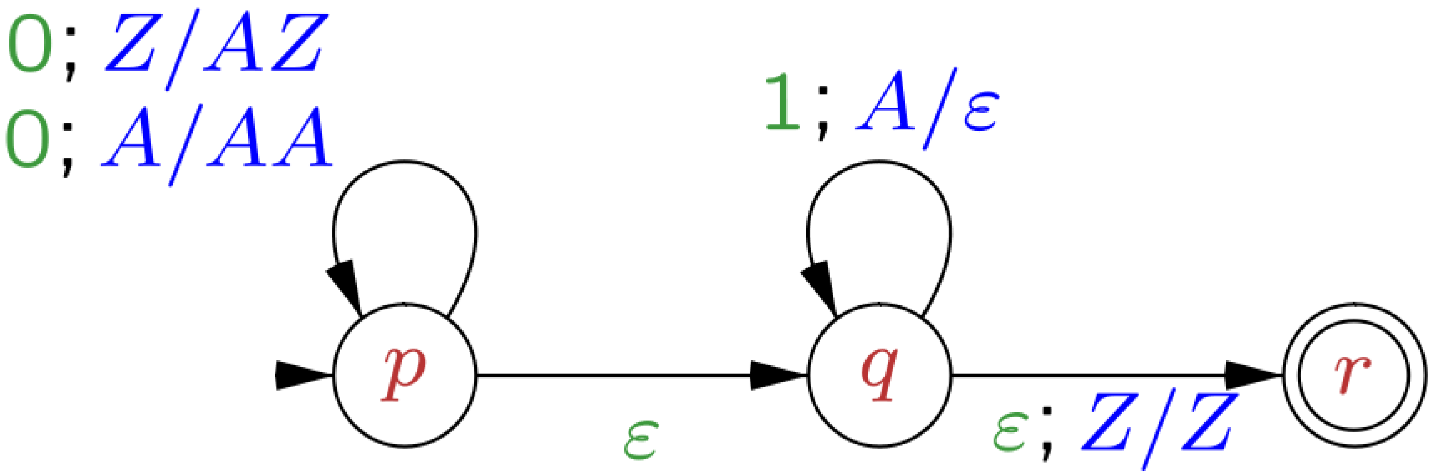
a start variable

S

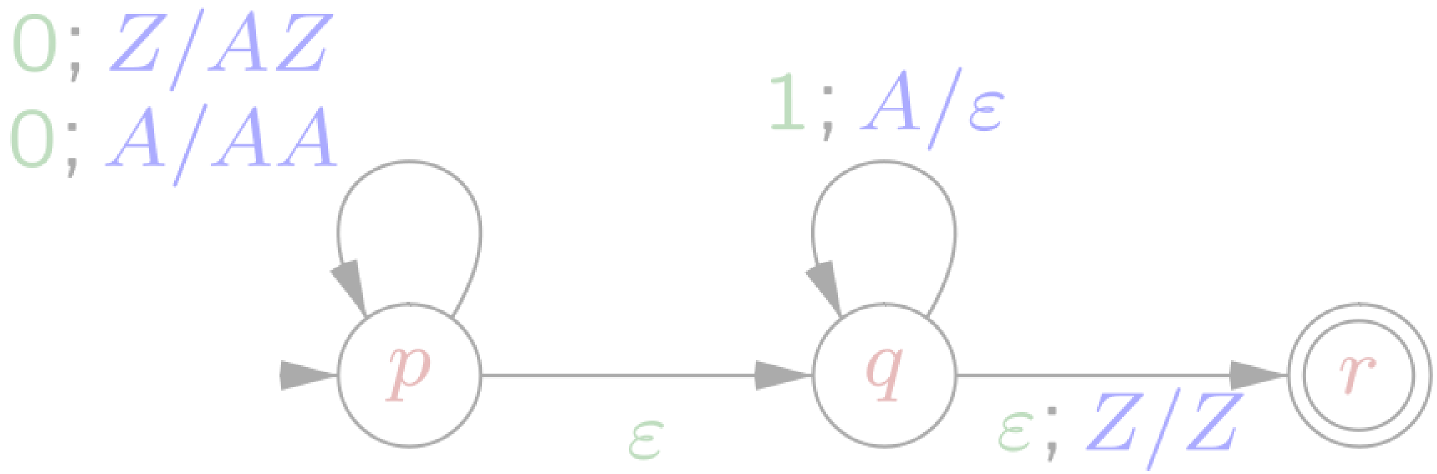
PDA

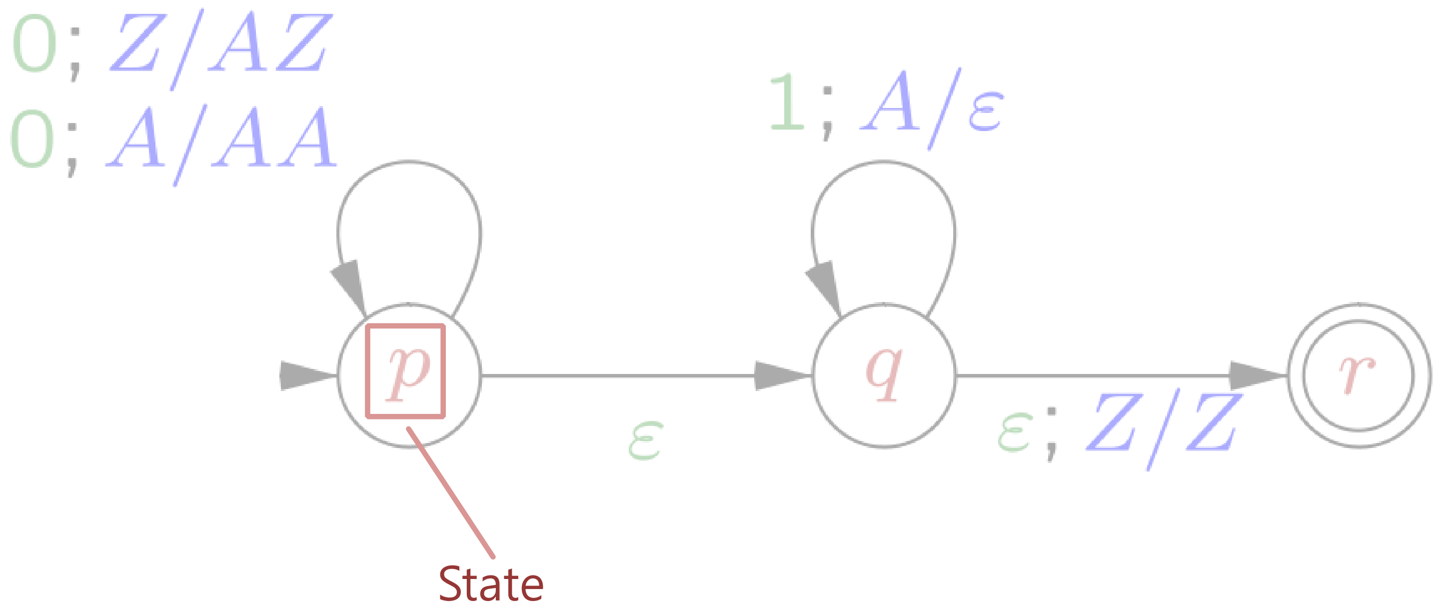
Push Down Automata

Pushdown Automata

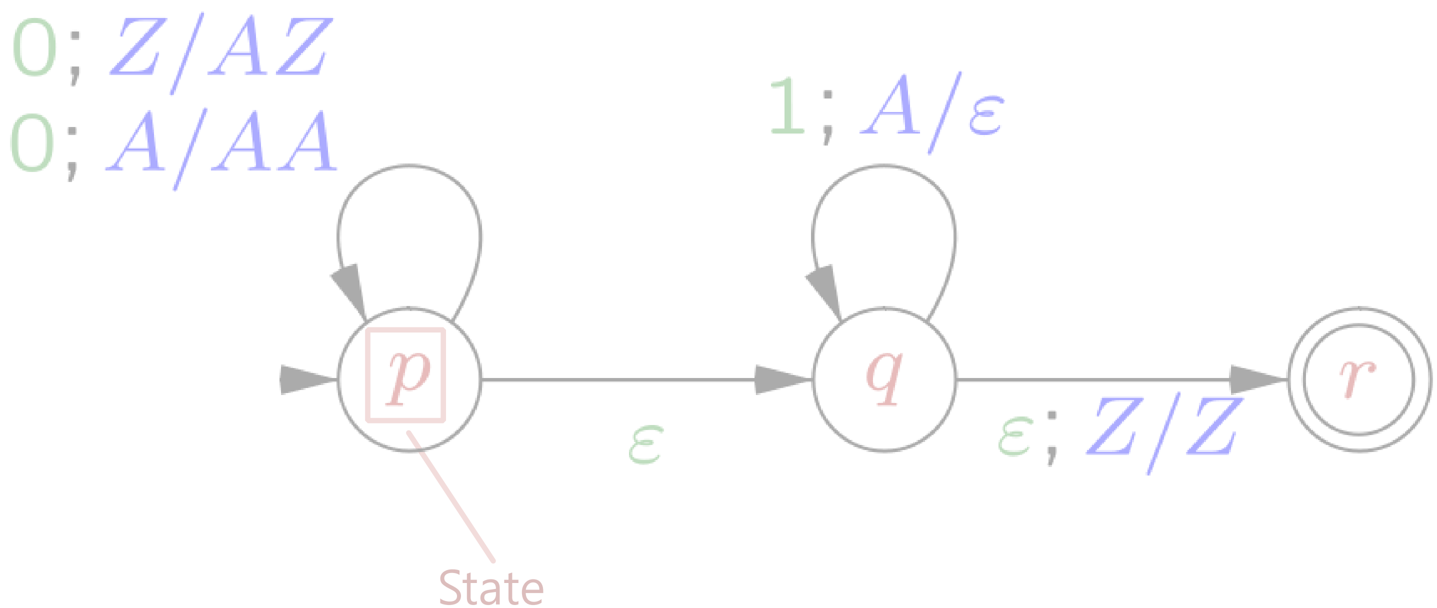


0; Z/AZ
0; A/AA
1; A/ε
p
ε
q
ε; Z/Z
r

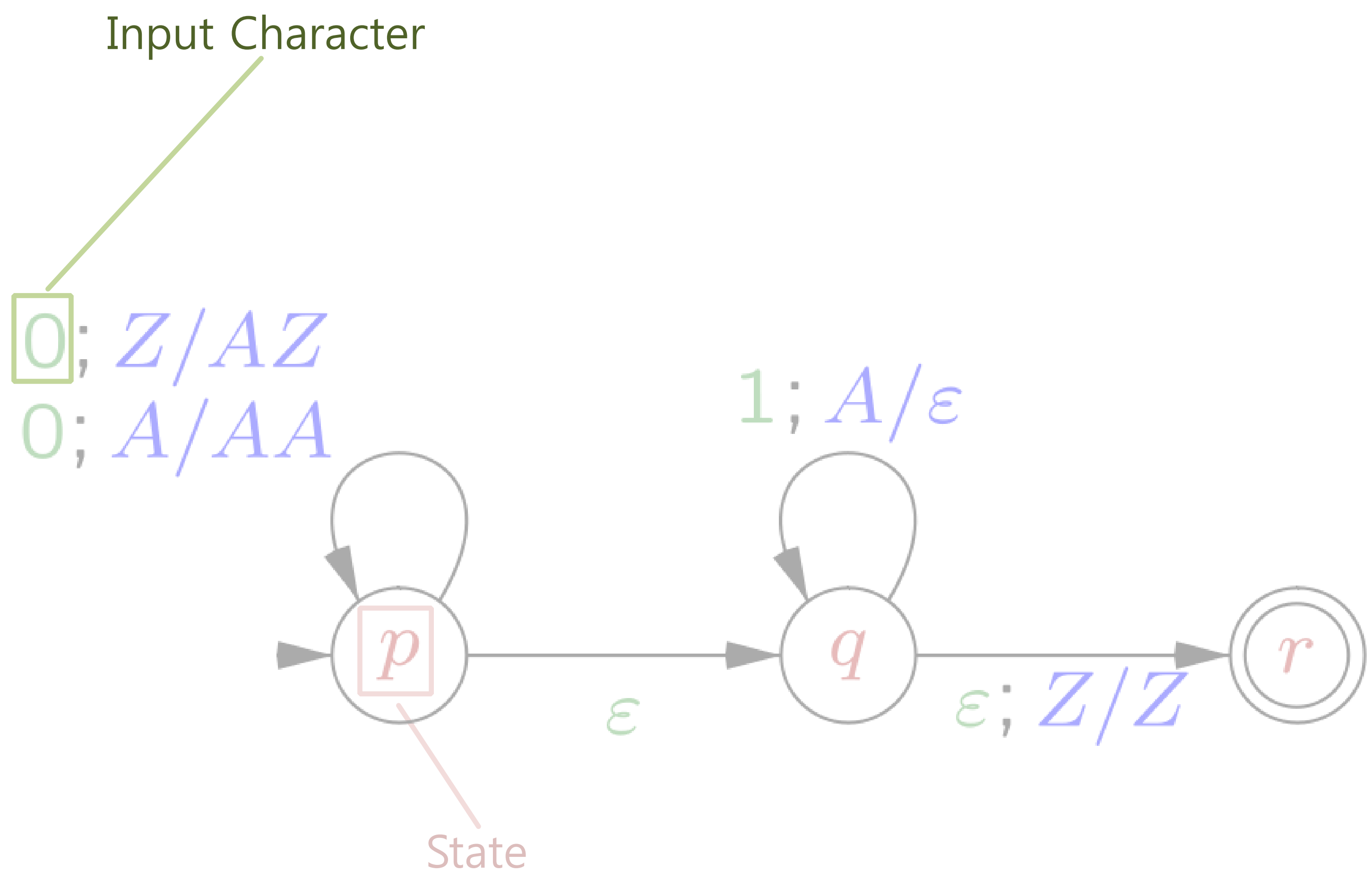

0; Z/AZ
0; A/AA
1; A/ε
p
q
r
ε
ε; Z/Z

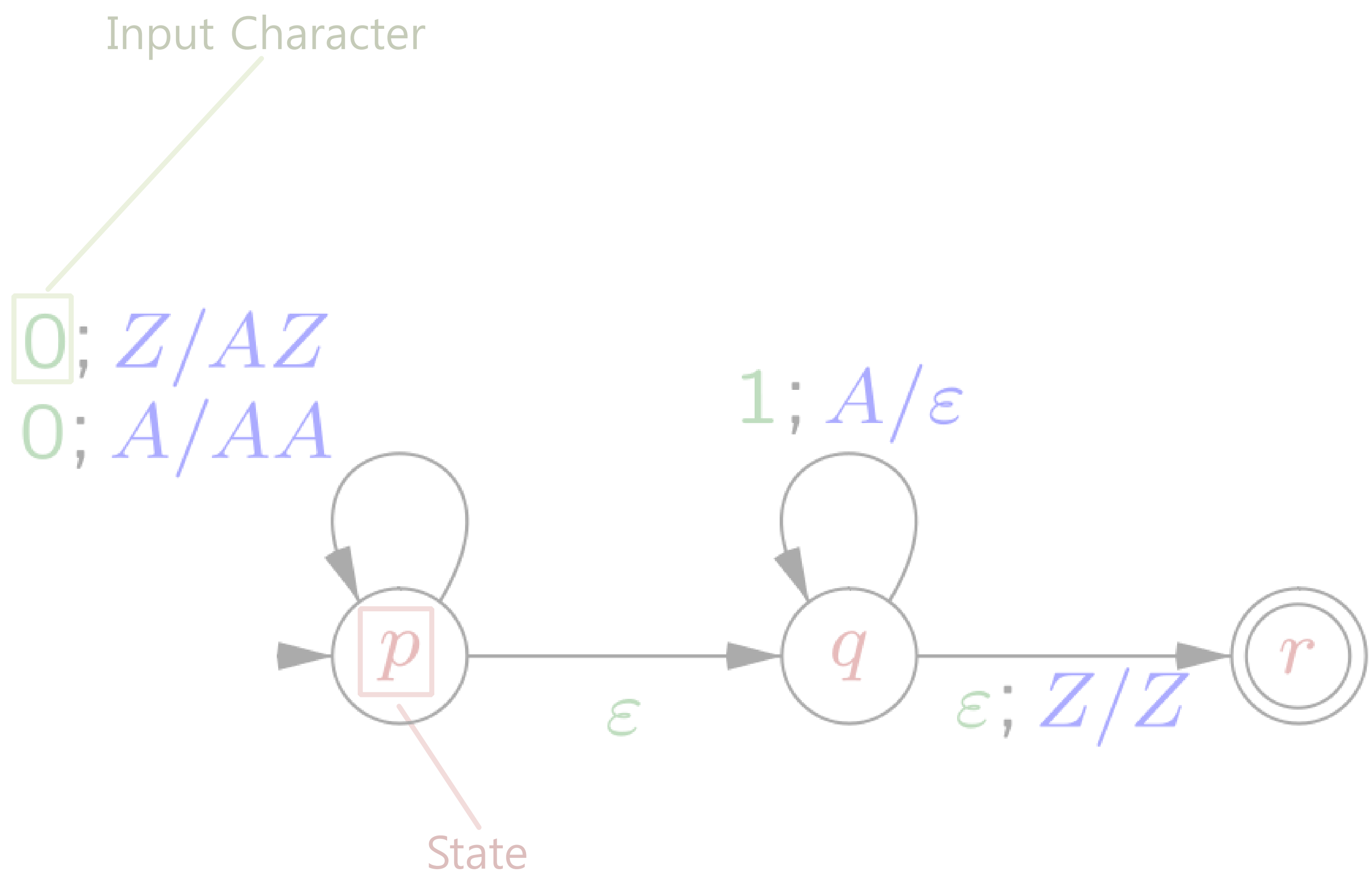


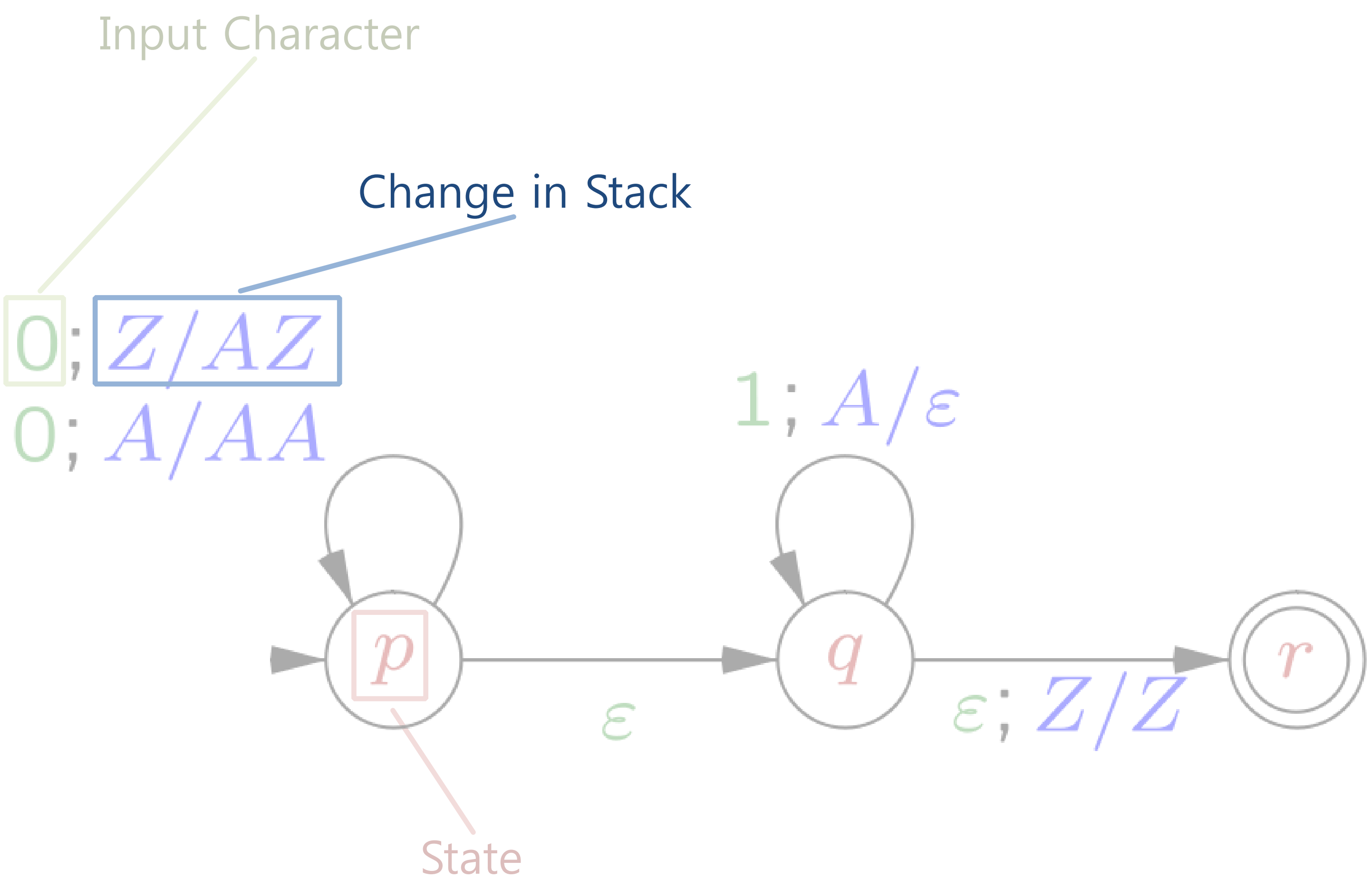
0; Z/AZ
0; A/AA
1; A/ε
p
q
r
ε
ε; Z/Z
State



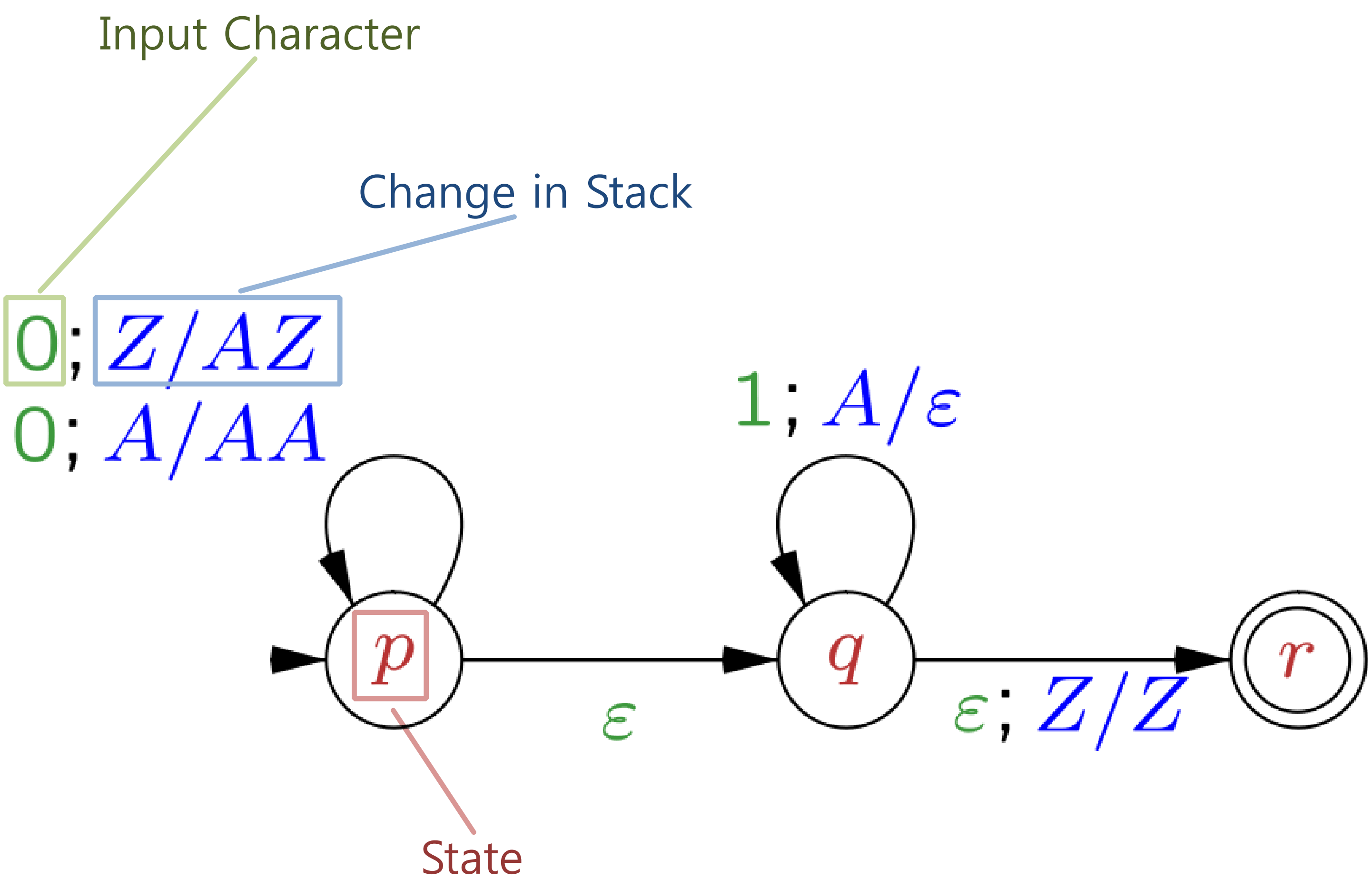
0; Z/AZ
0; A/AA
1; A/ε
p
q
r
ε
ε; Z/Z
State


Input Character
0; Z/AZ
0; A/AA
1; A/ε
p
ε
q
ε; Z/Z
r
State


Input Character
0; Z/AZ
0; A/AA
1; A/ε
p
ε
q
ε; Z/Z
r
State



Input Character
Change in Stack
0; Z/AZ
0; A/AA
1; A/ε
p
ε
q
ε; Z/Z
r
State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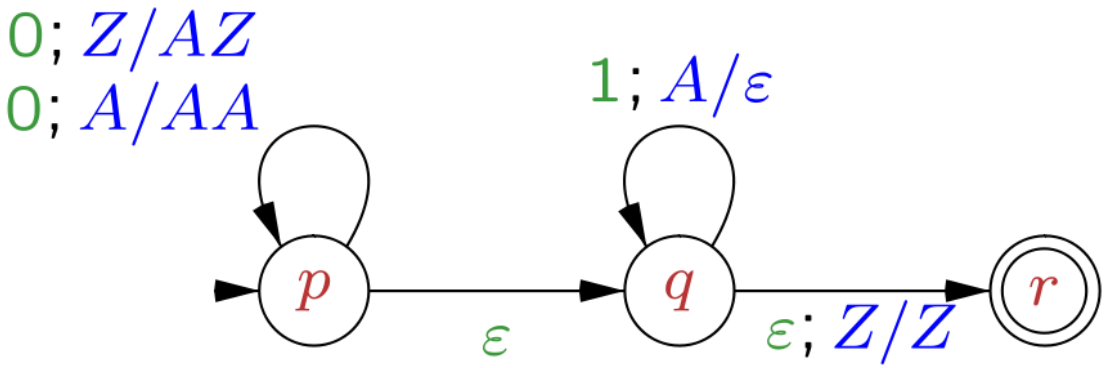

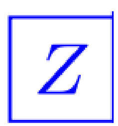

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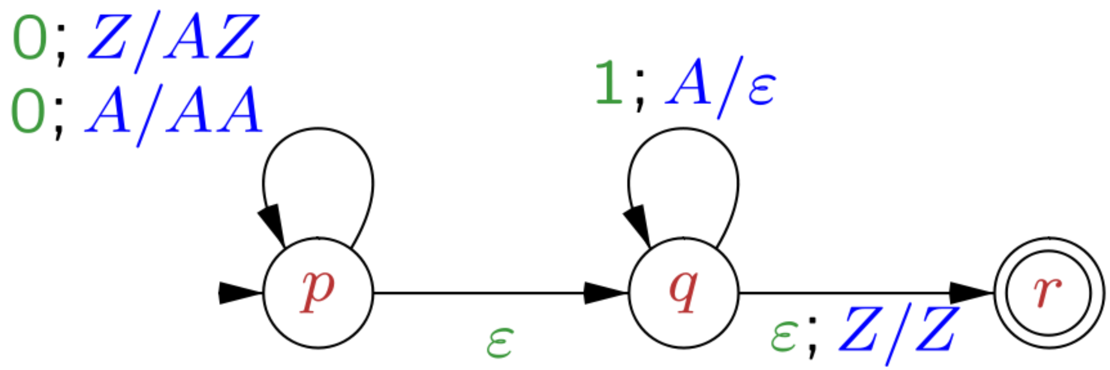
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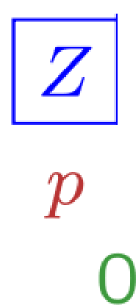
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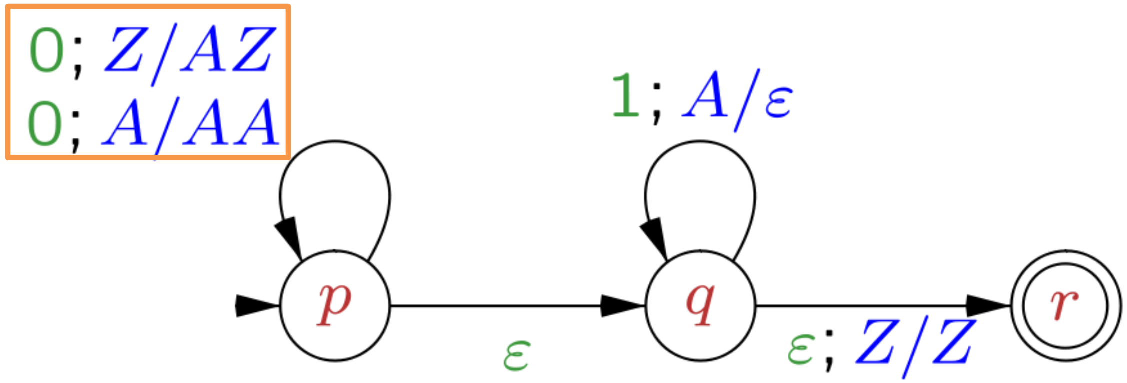
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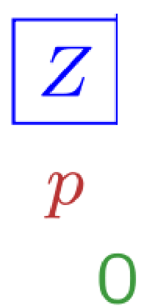
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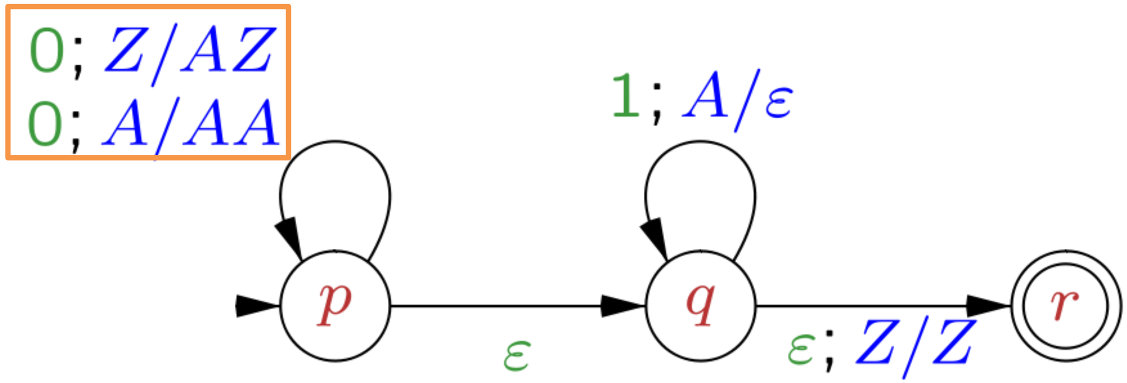
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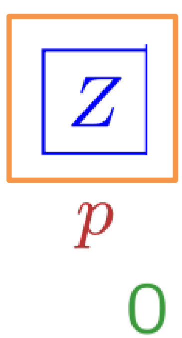
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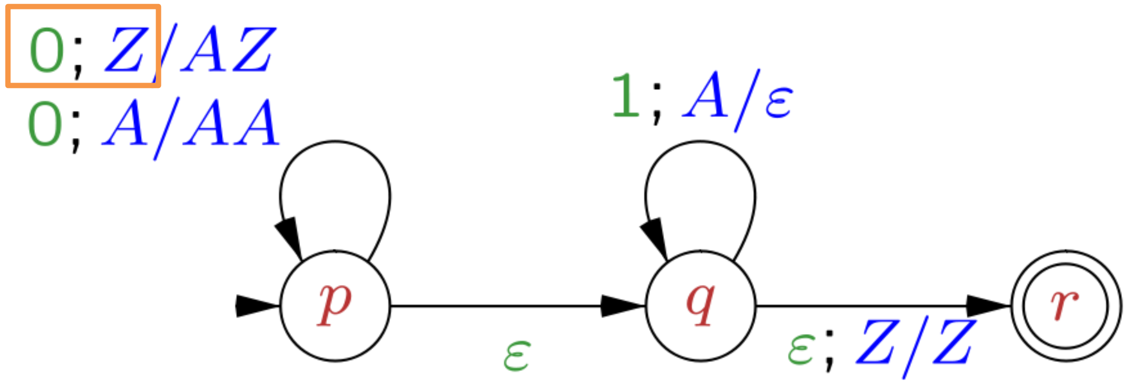
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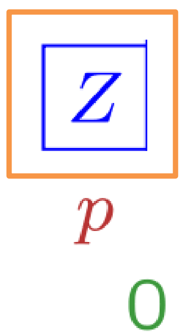
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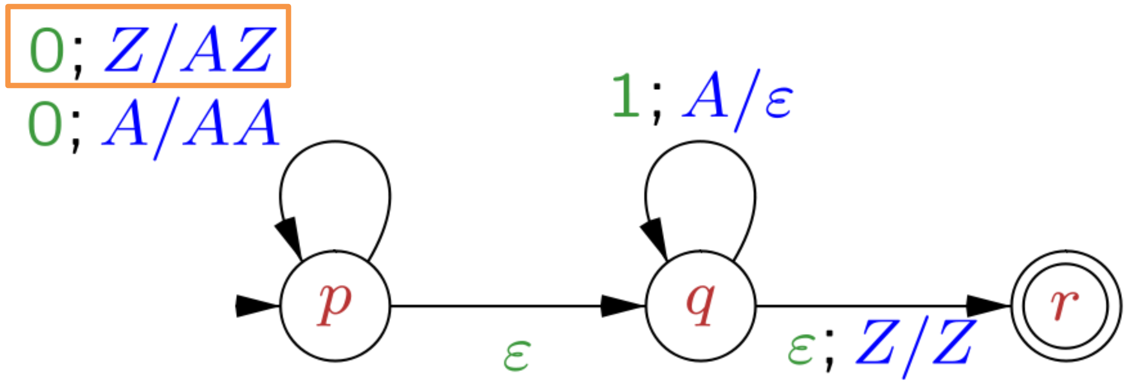
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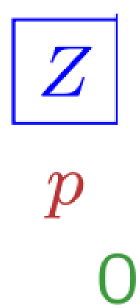
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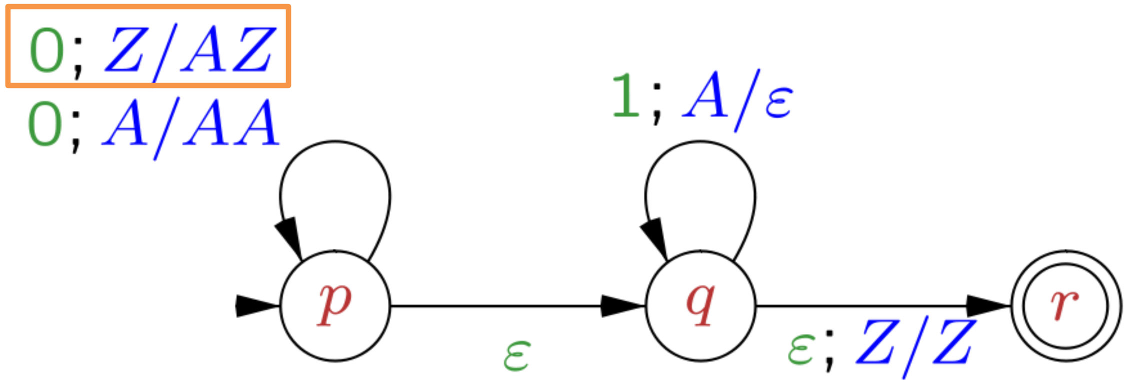
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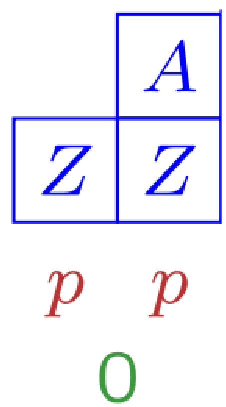
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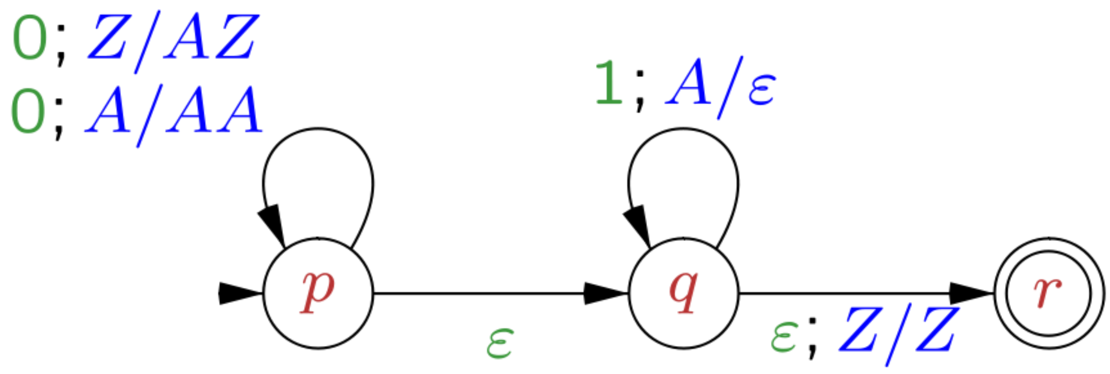

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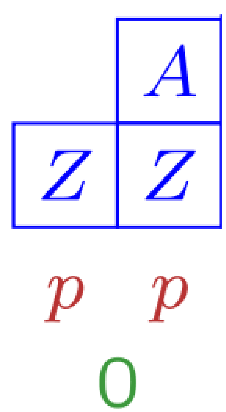

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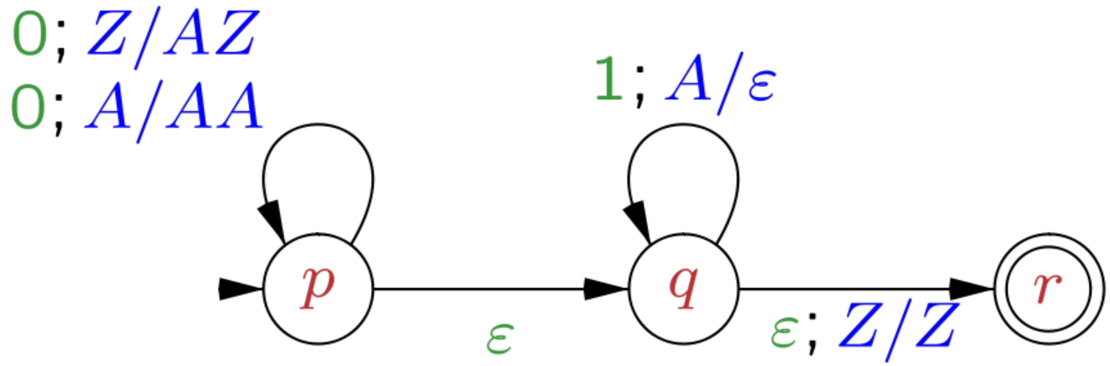
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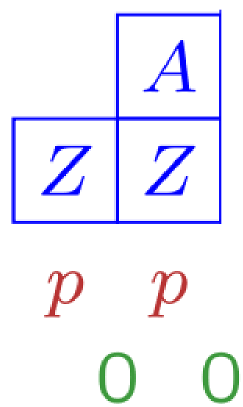
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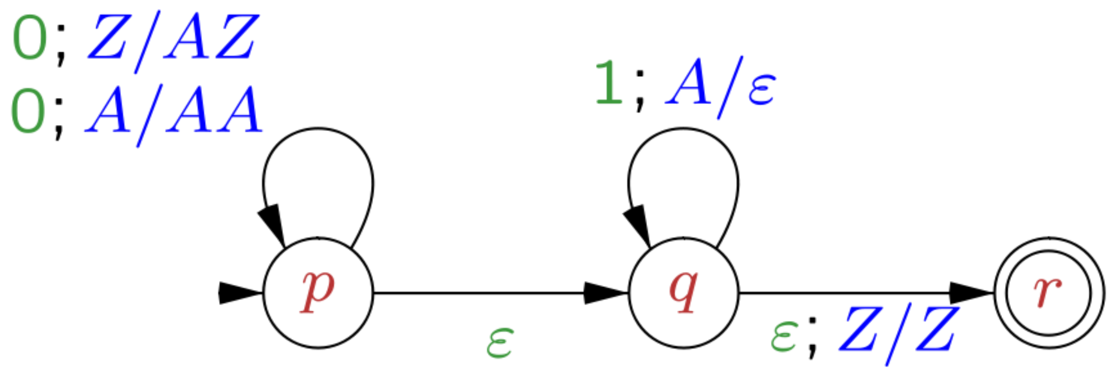

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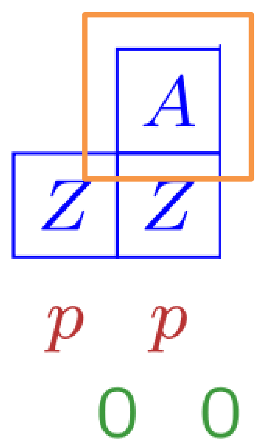

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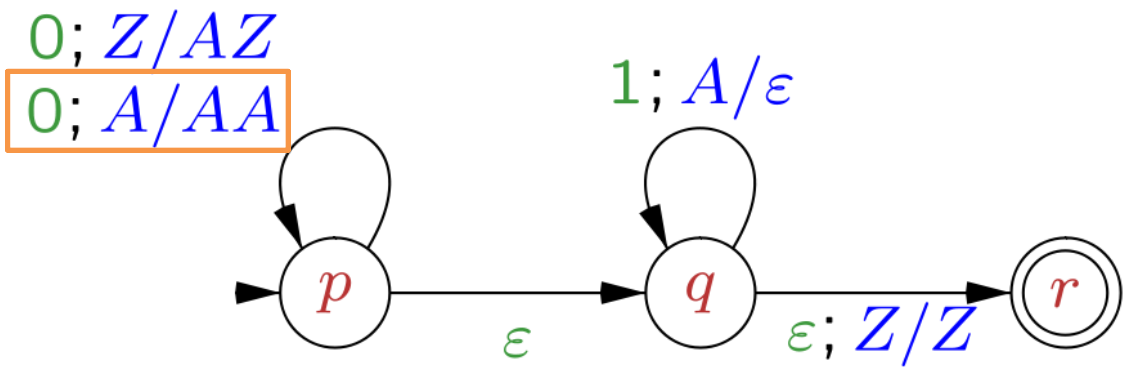

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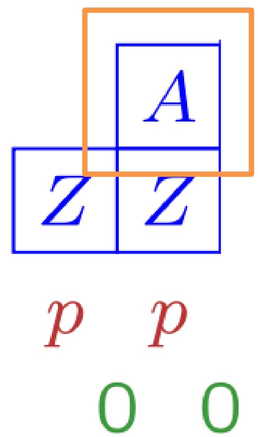

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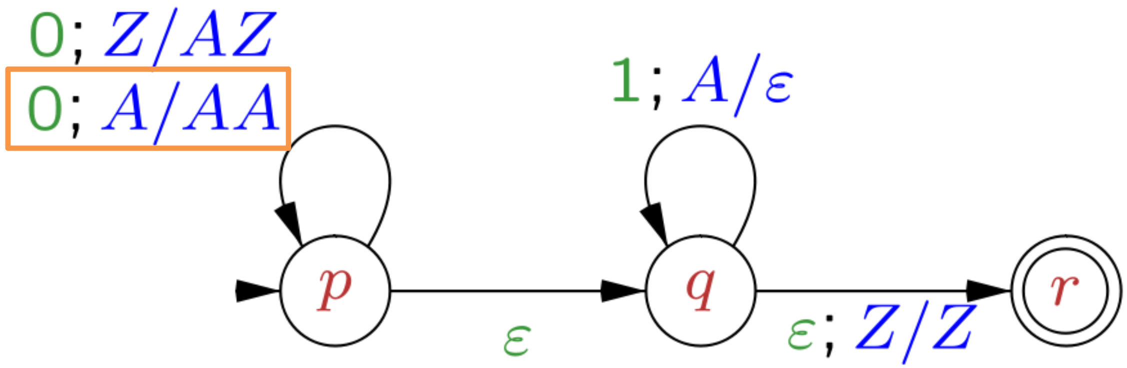
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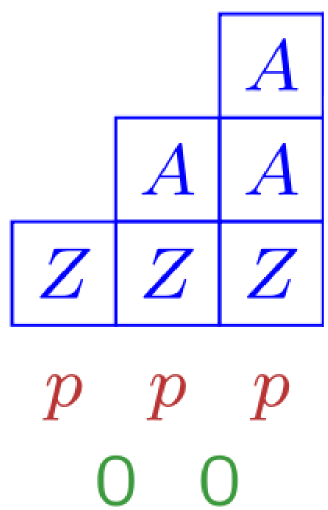
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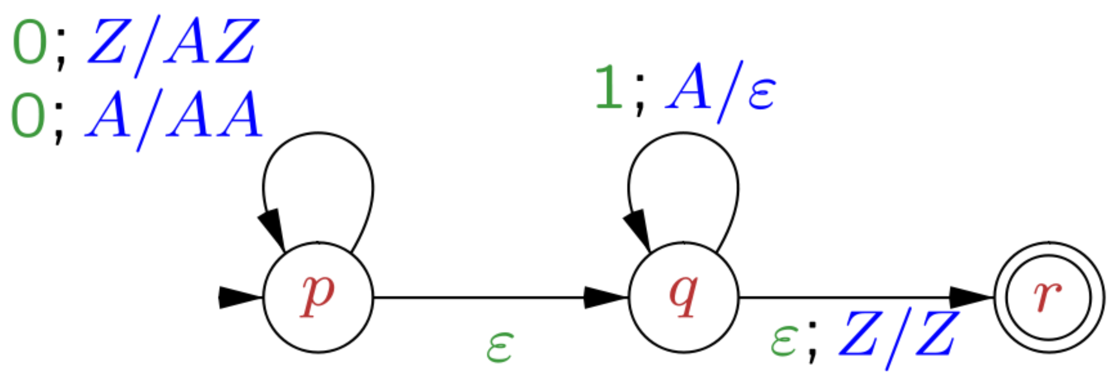
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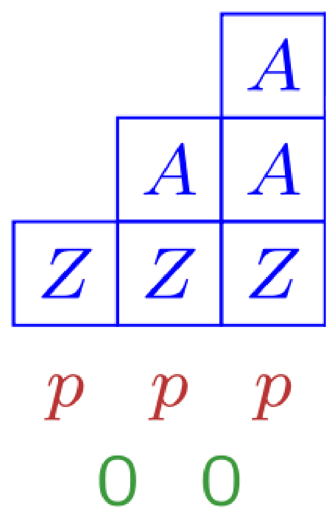
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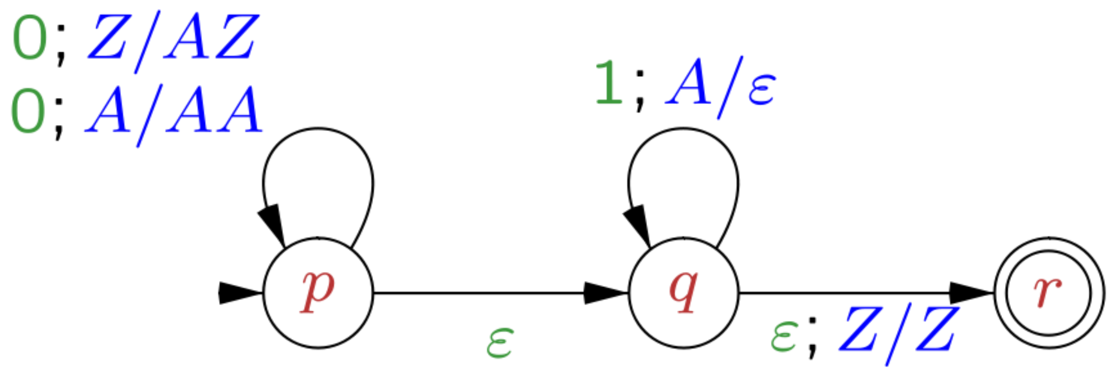
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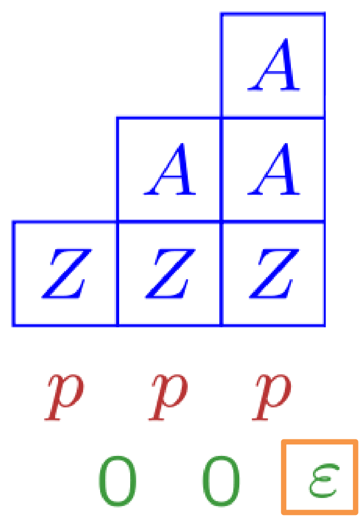
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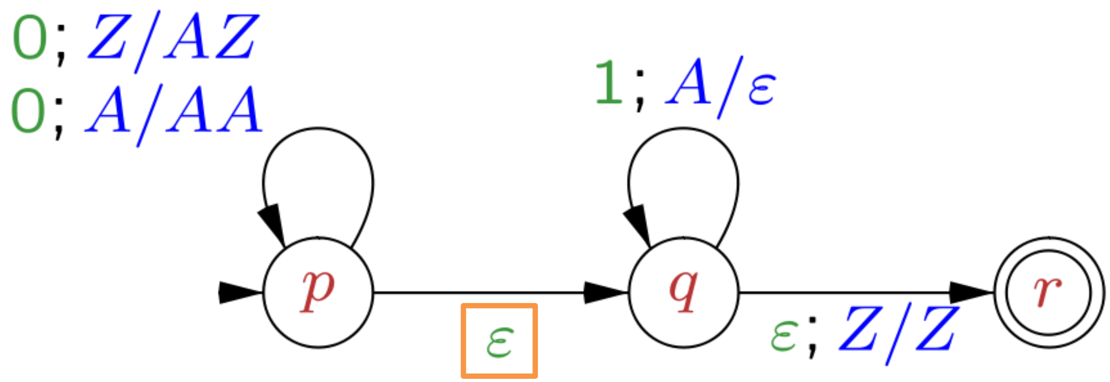
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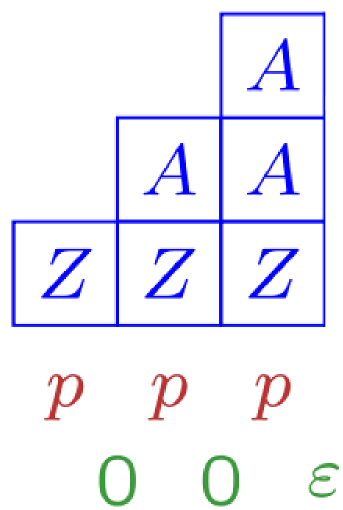
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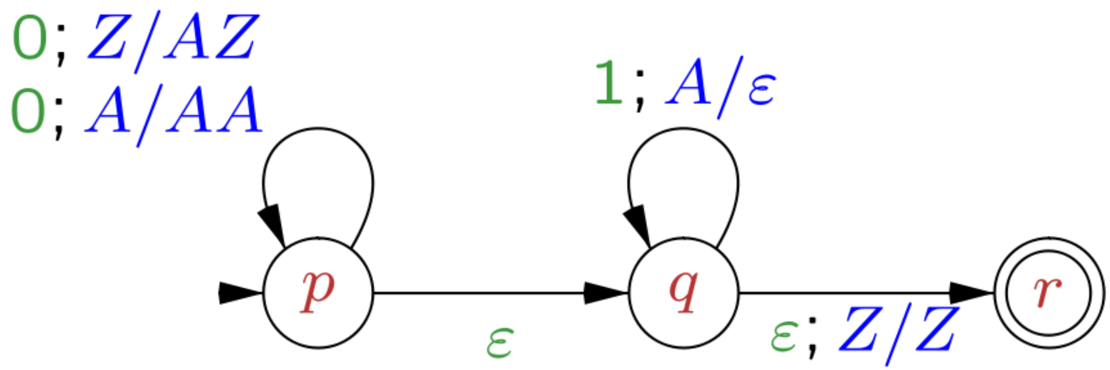

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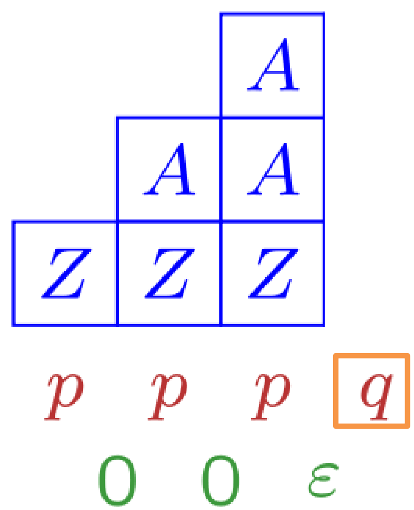

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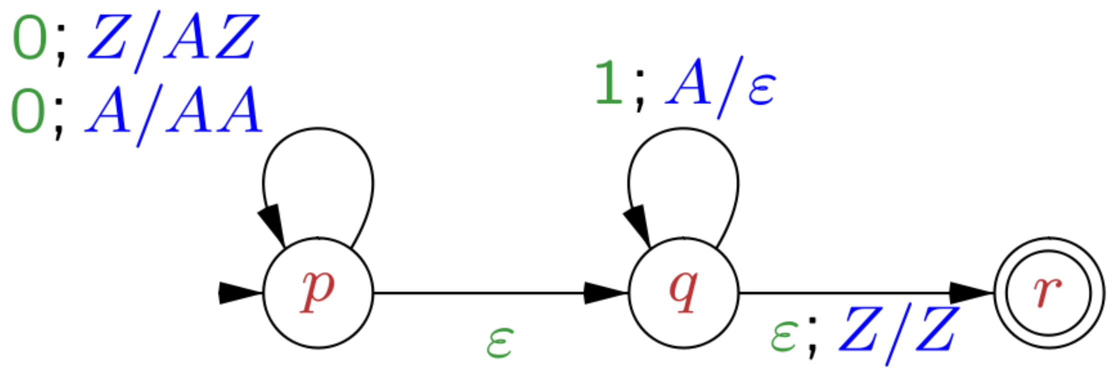
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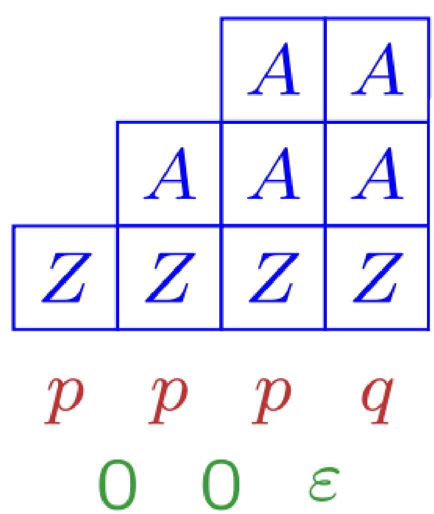
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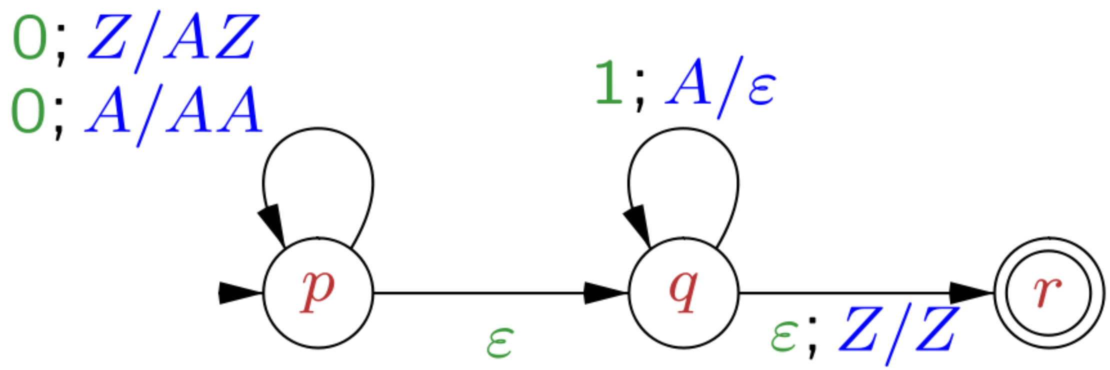
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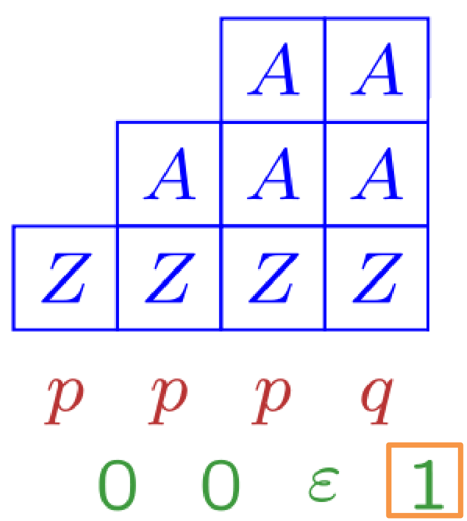
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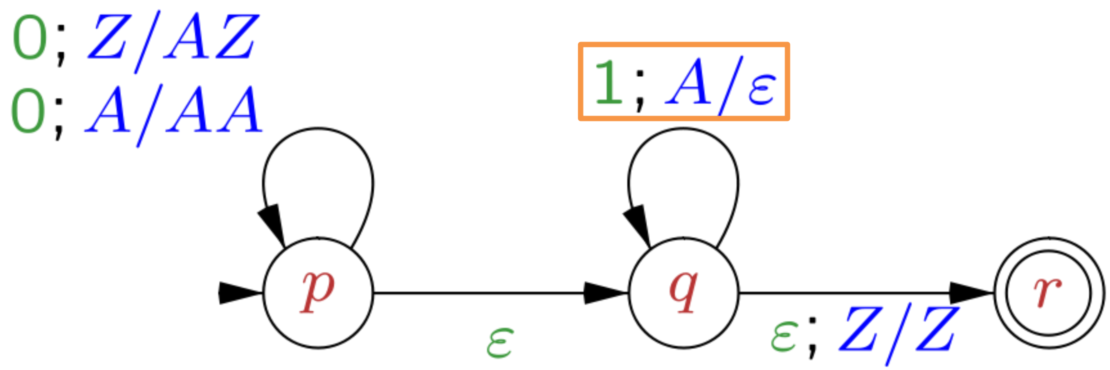

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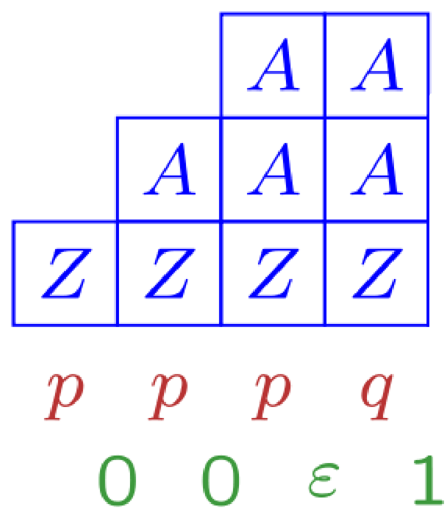

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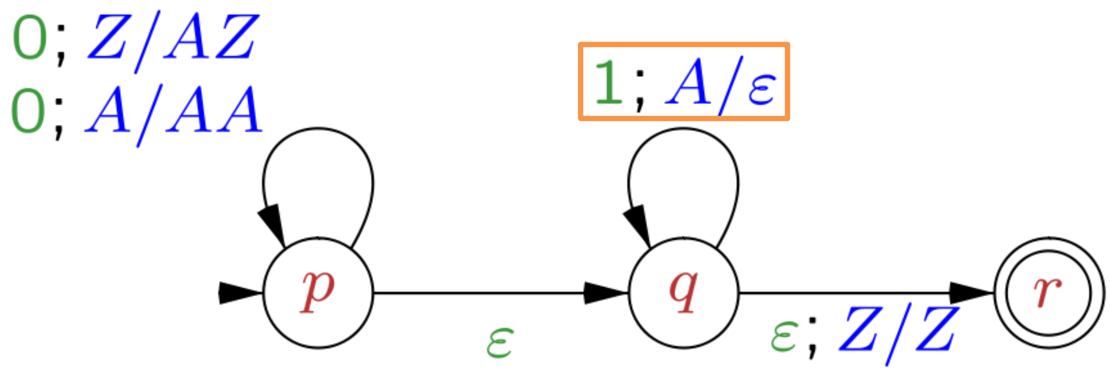
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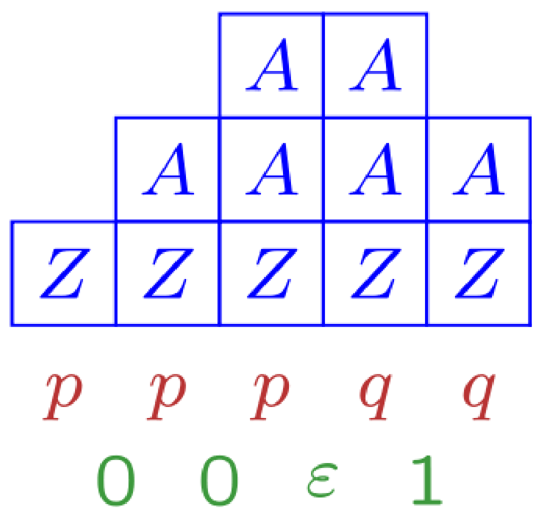
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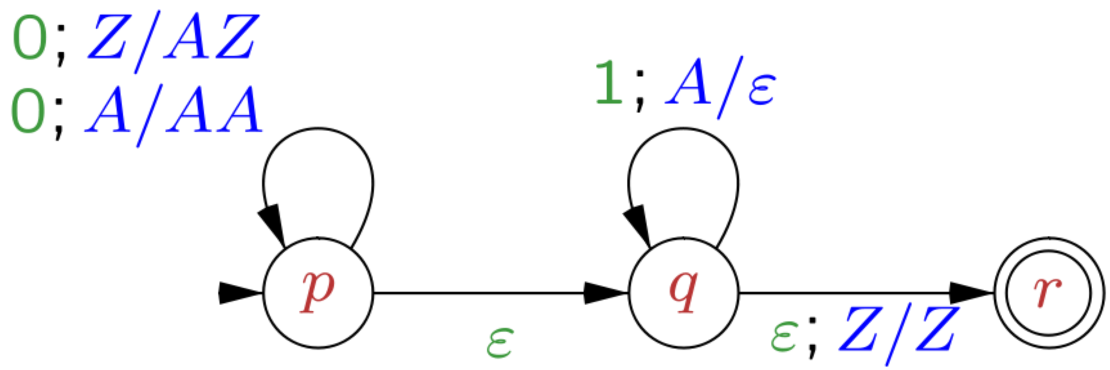
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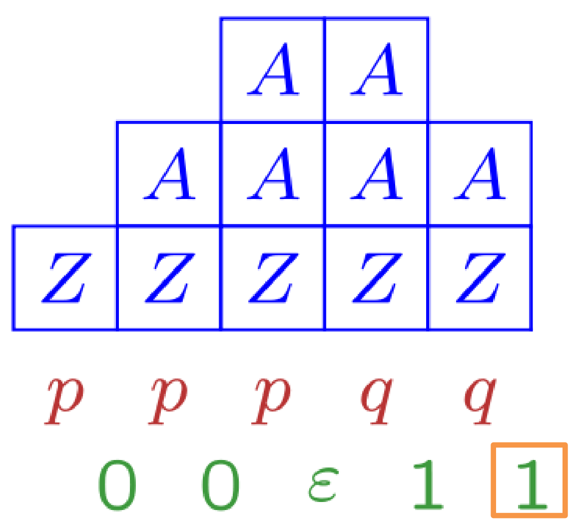
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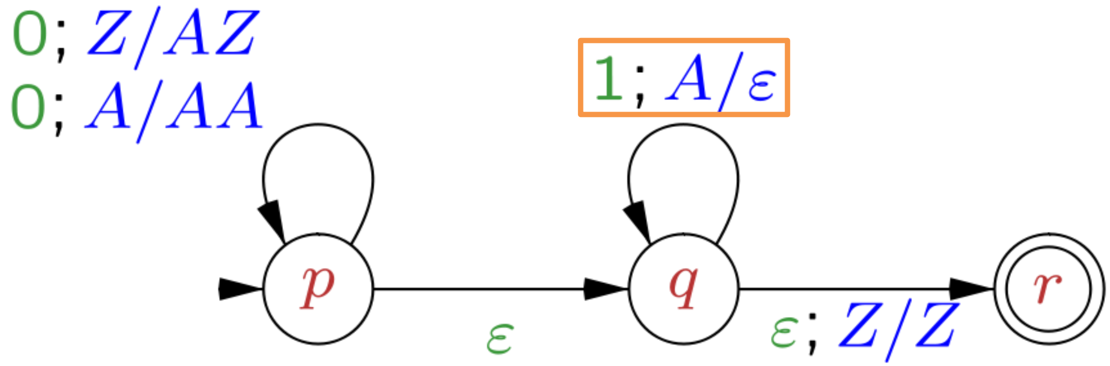
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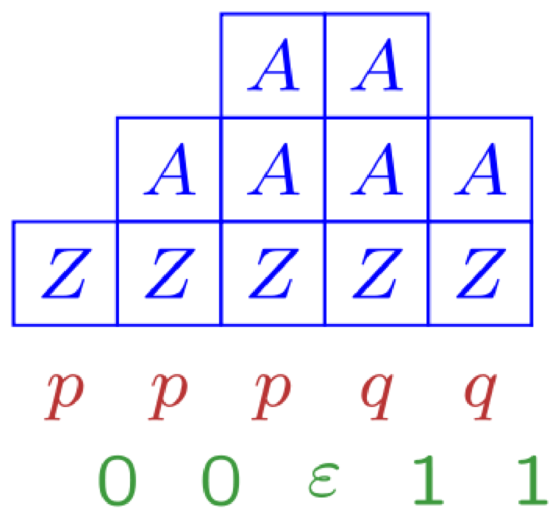
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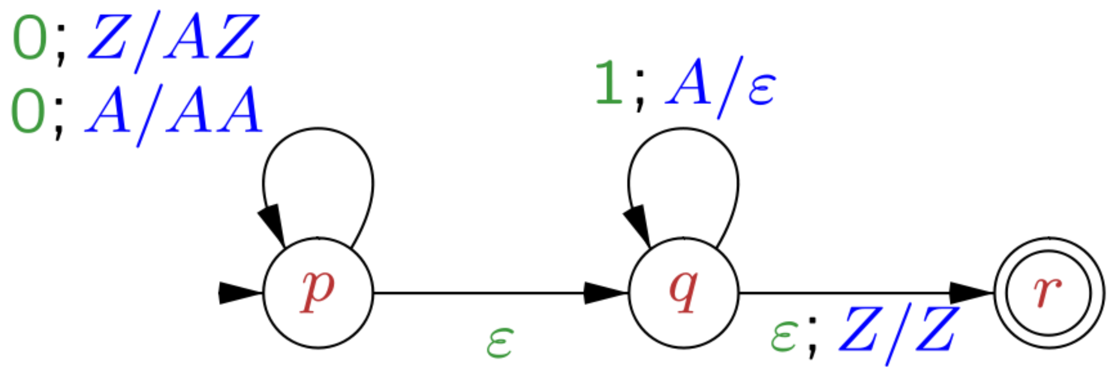
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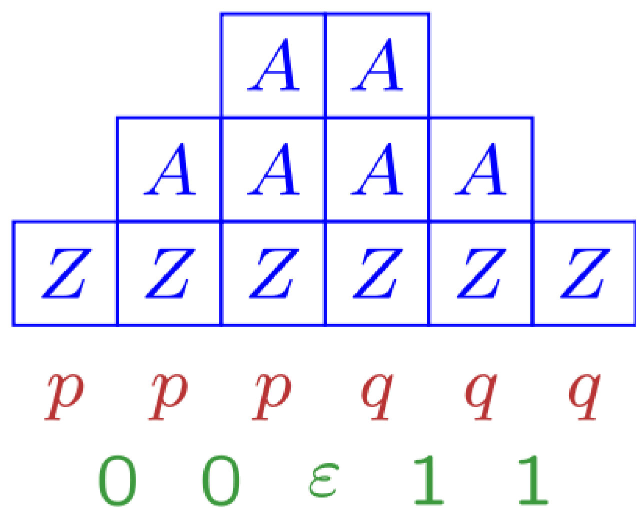
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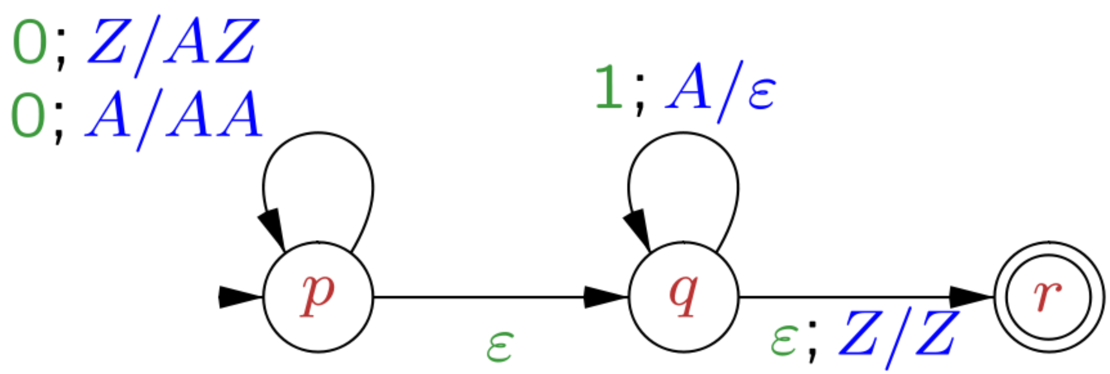
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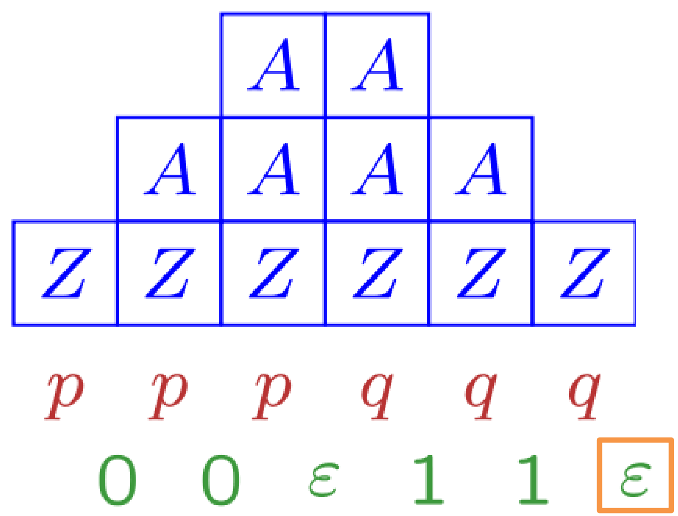
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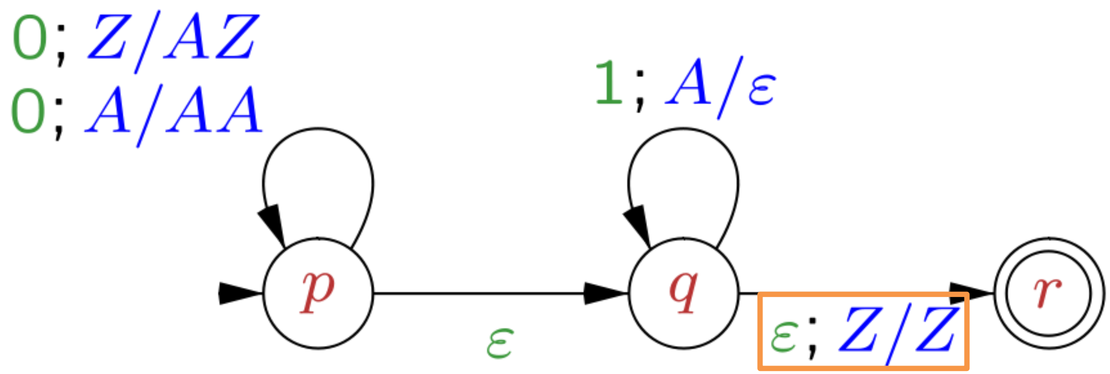
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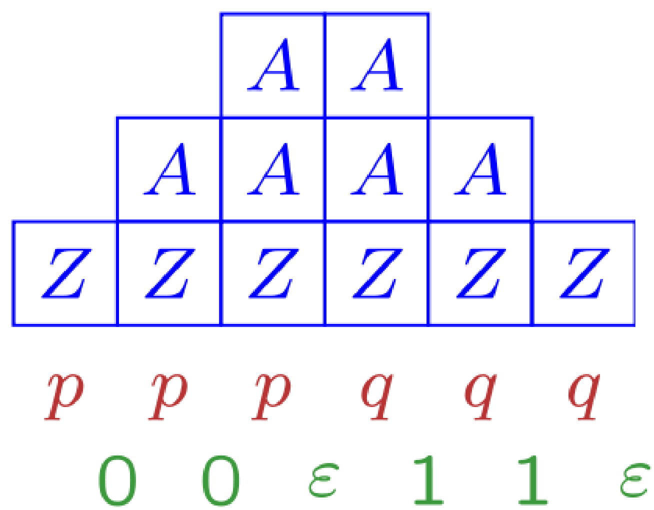
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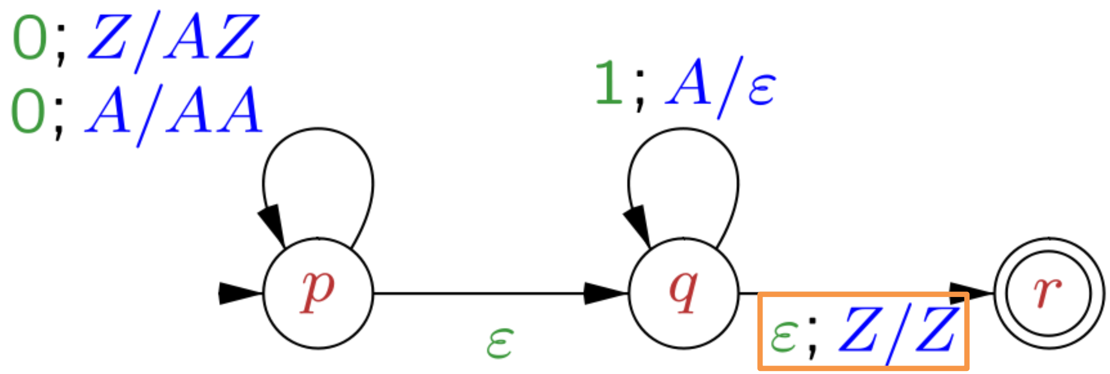
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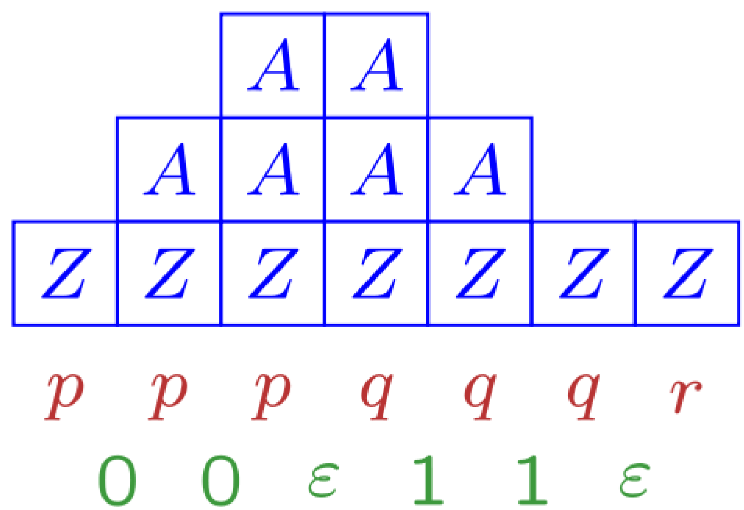
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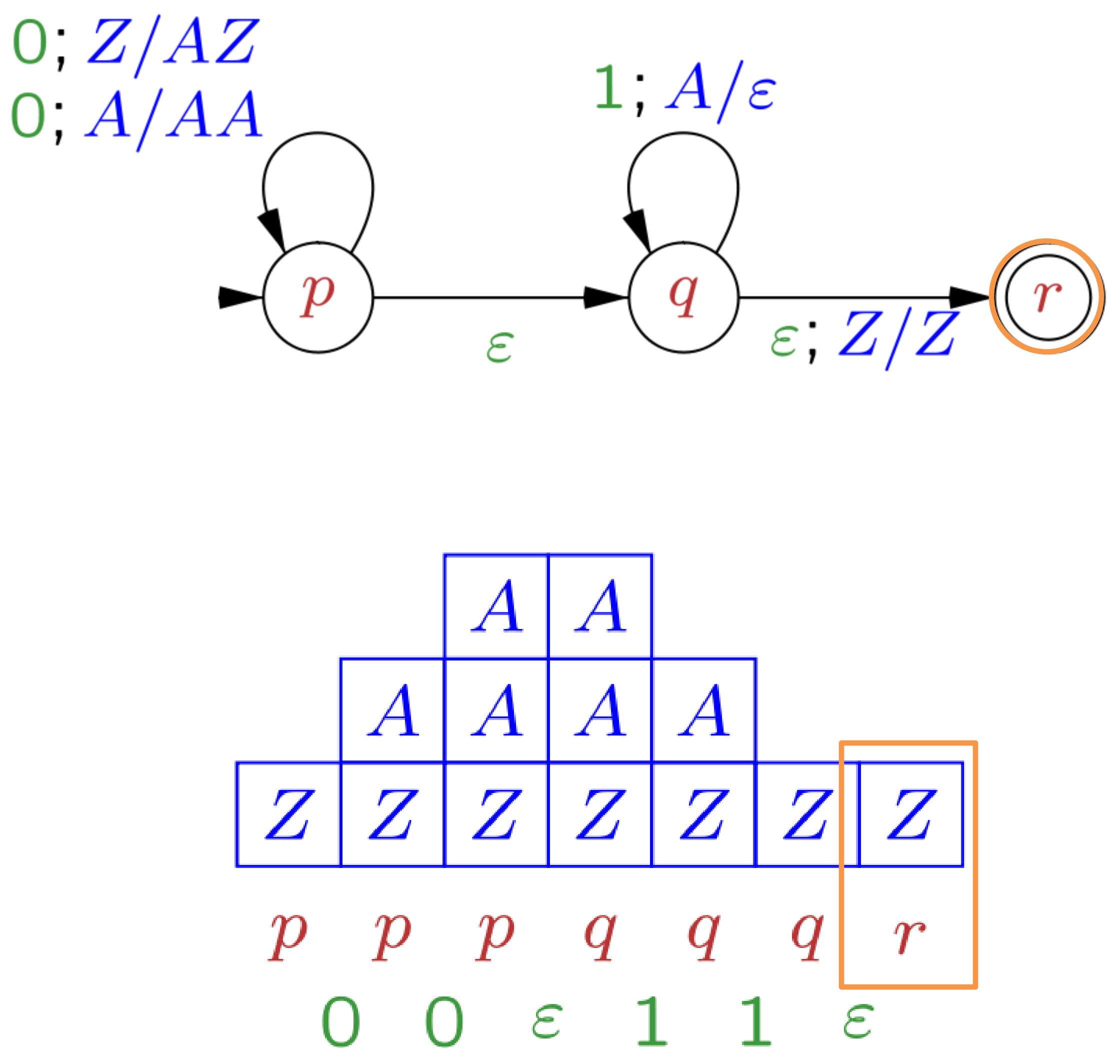

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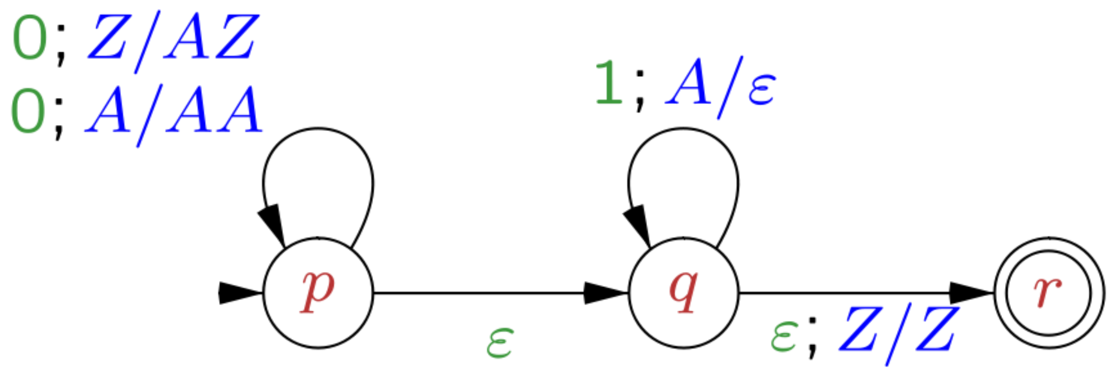
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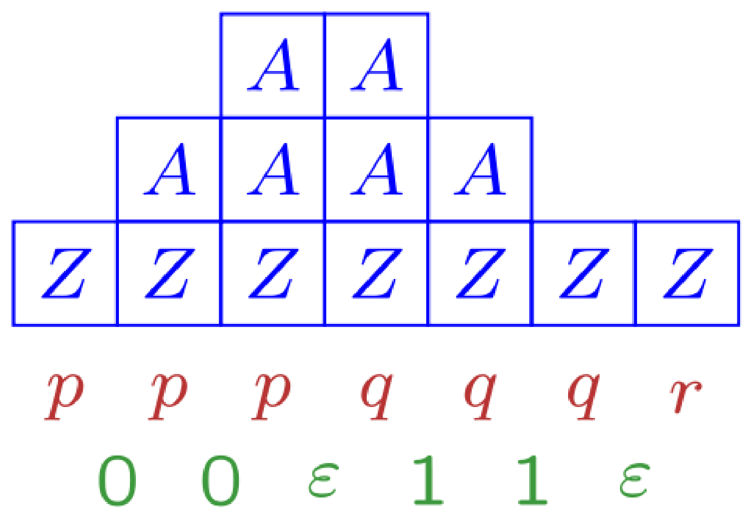
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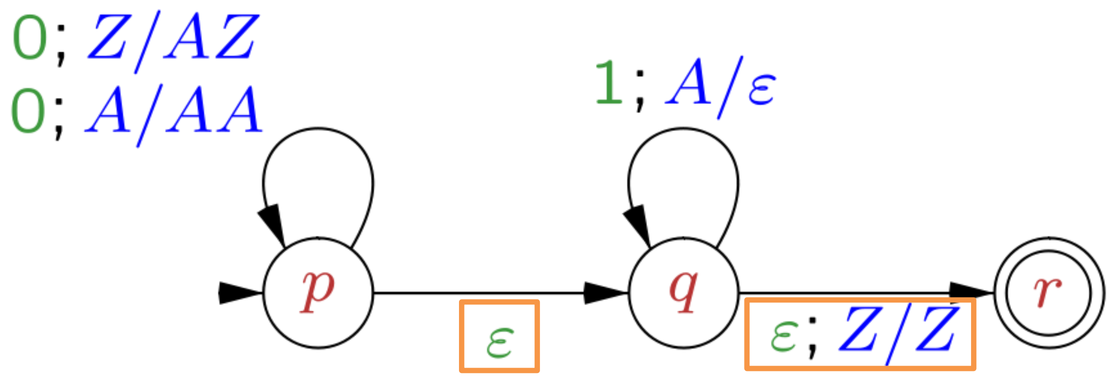
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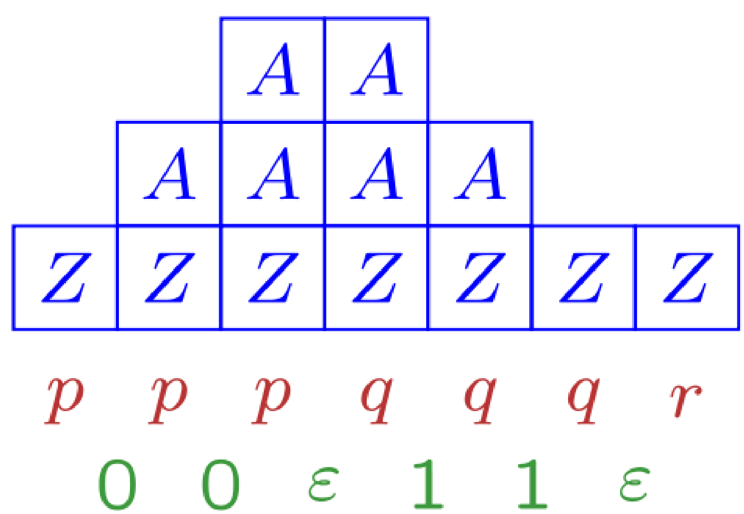
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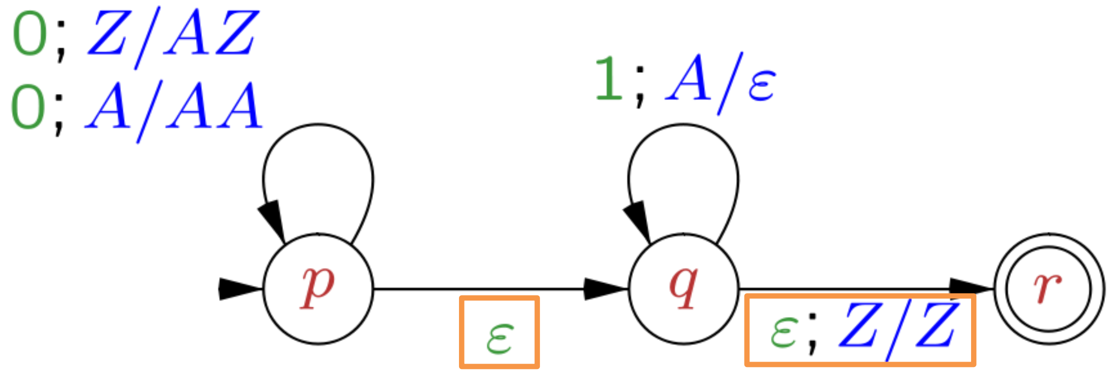

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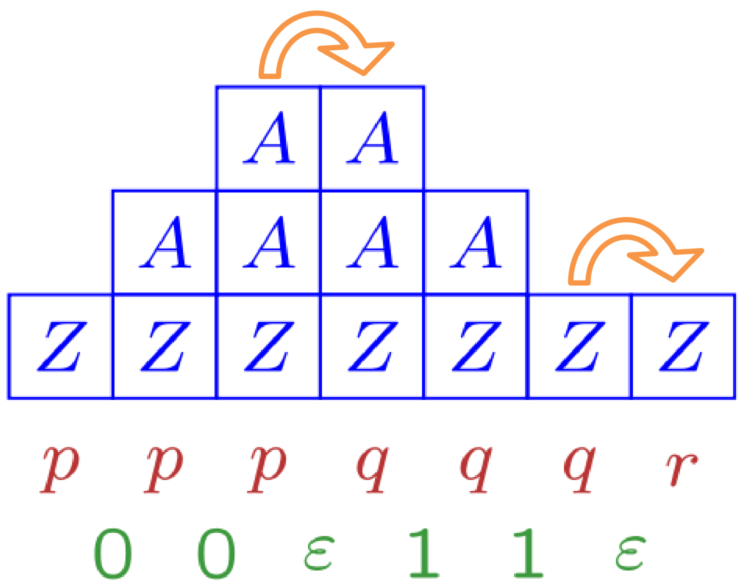

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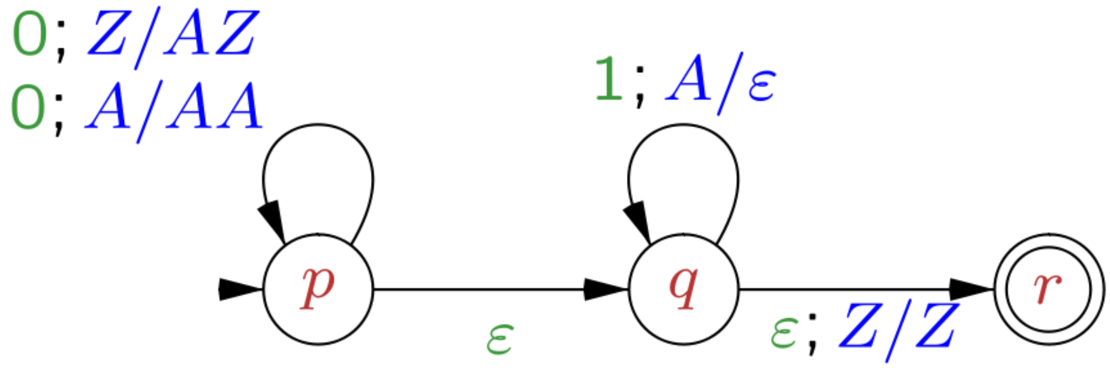
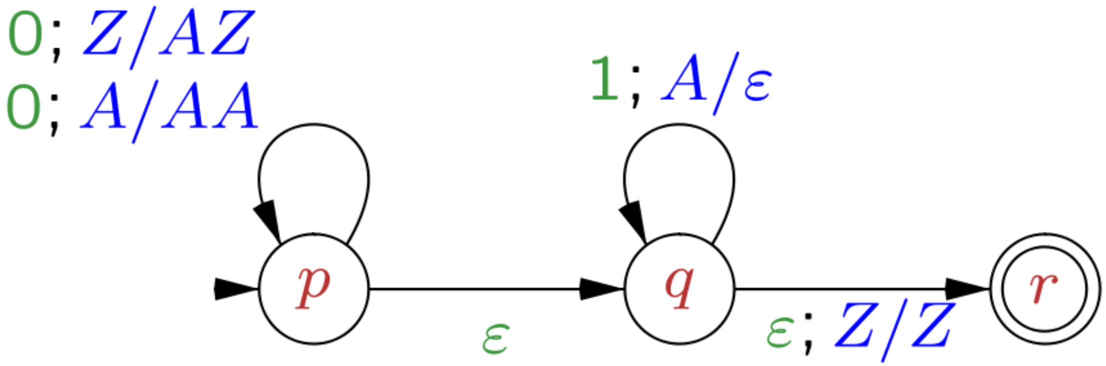
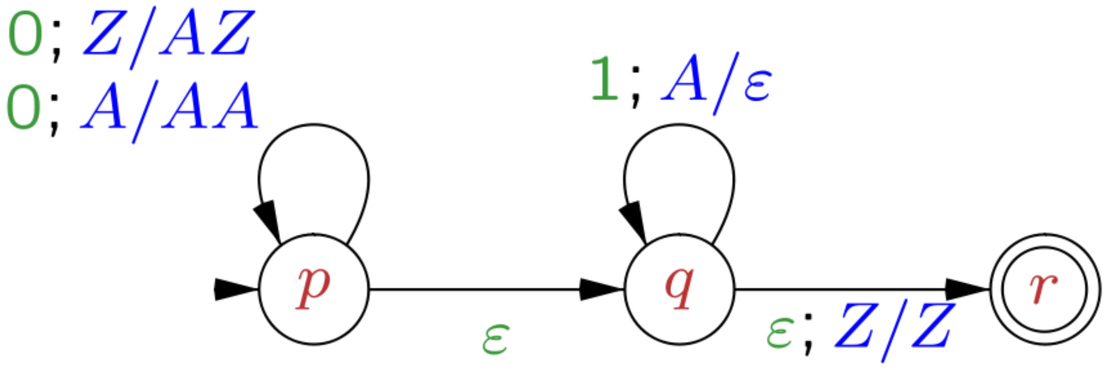
p p p q q q r
0 0 ε 1 1 ε



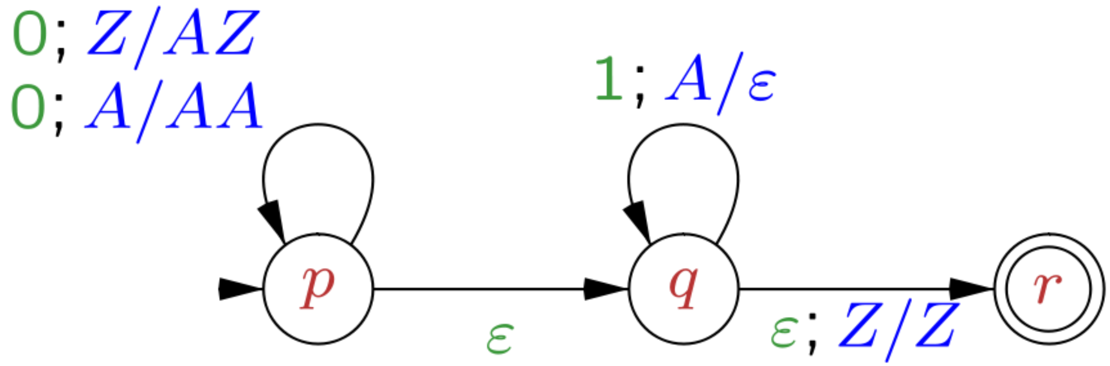
$(p,\ 0011,\qquad Z)\ \vdash$



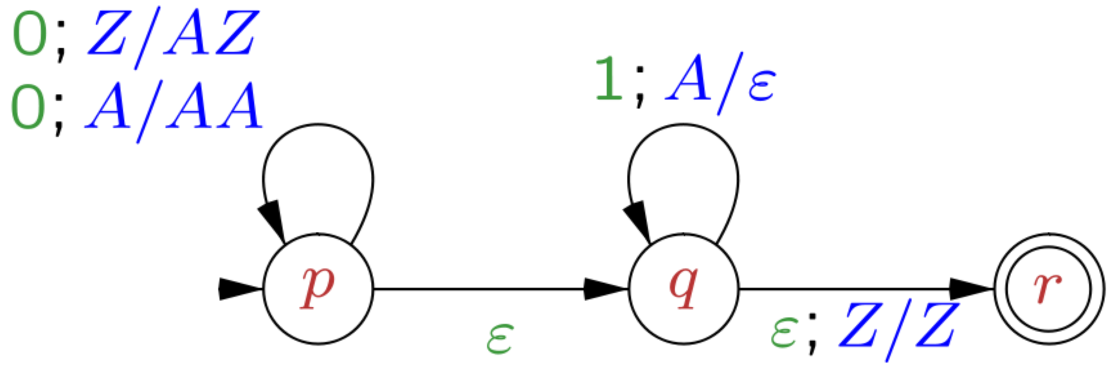
$$(p,\ 0011,\ Z) \vdash$$
$$(p,\ 011,\ AZ) \vdash$$



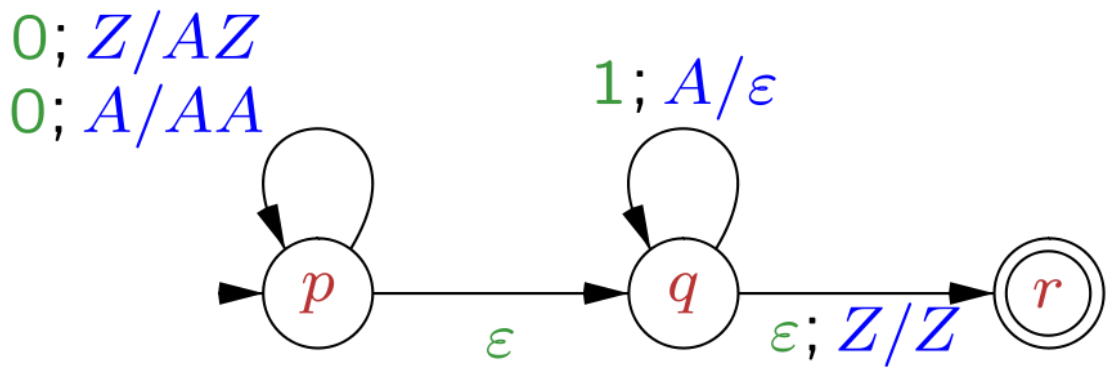
$$
\begin{array}{rl}
(p,\ 0011,\ Z) & \vdash \\
(p,\ 011,\ AZ) & \vdash \\
(p,\ 11,\ AAZ) & \vdash
\end{array}
$$



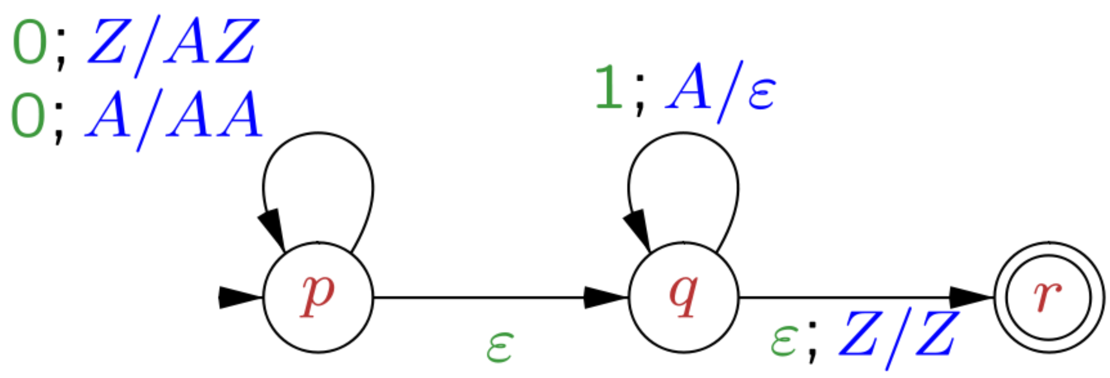
$$
\begin{aligned}
(p,\ 0011,\ Z) &\vdash \\
(p,\ 011,\ AZ) &\vdash \\
(p,\ 11,\ AAZ) &\vdash \\
(q,\ 11,\ AAZ) &\vdash
\end{aligned}
$$



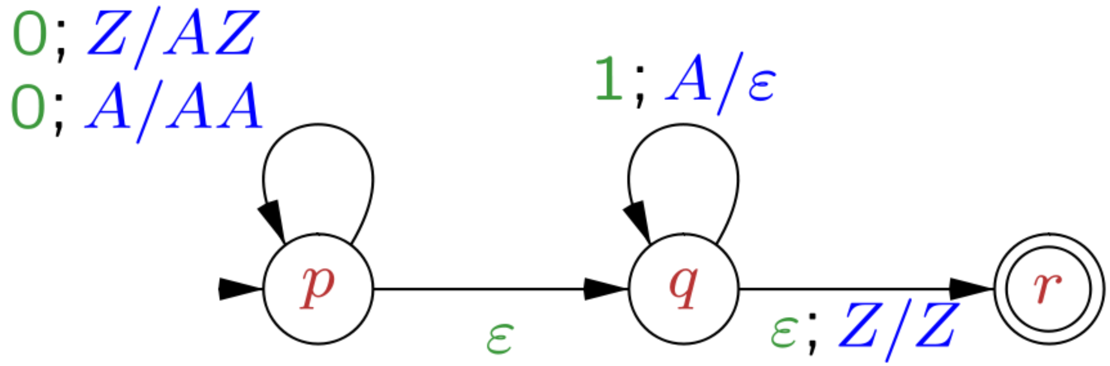
$$
\begin{array}{l}
(p,\ 0011,\ Z)\ \vdash \\
(p,\ 011,\ AZ)\ \vdash \\
(p,\ 11,\ AAZ)\ \vdash \\
(q,\ 11,\ AAZ)\ \vdash \\
(q,\ 1,\ AZ)\ \vdash
\end{array}
$$



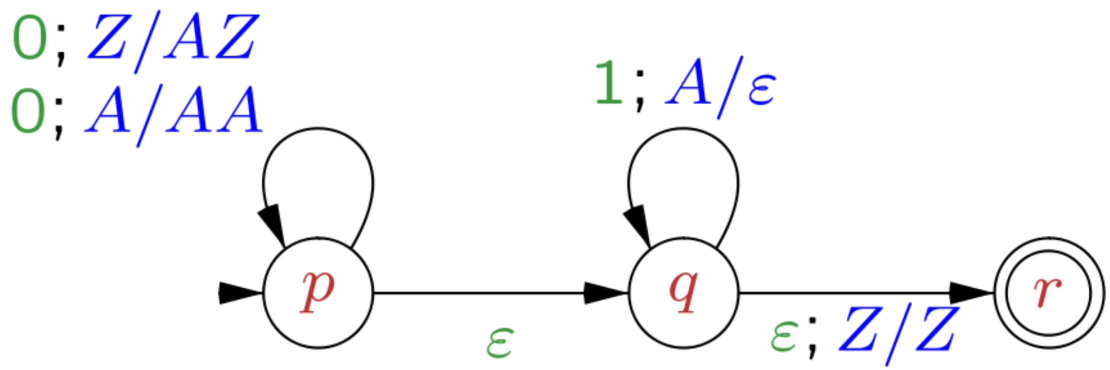
$$
\begin{aligned}
(p,\ 0011,\ Z) &\vdash \\
(p,\ 011,\ AZ) &\vdash \\
(p,\ 11,\ AAZ) &\vdash \\
(q,\ 11,\ AAZ) &\vdash \\
(q,\ 1,\ AZ) &\vdash \\
(q,\ \varepsilon,\ Z) &\vdash
\end{aligned}
$$



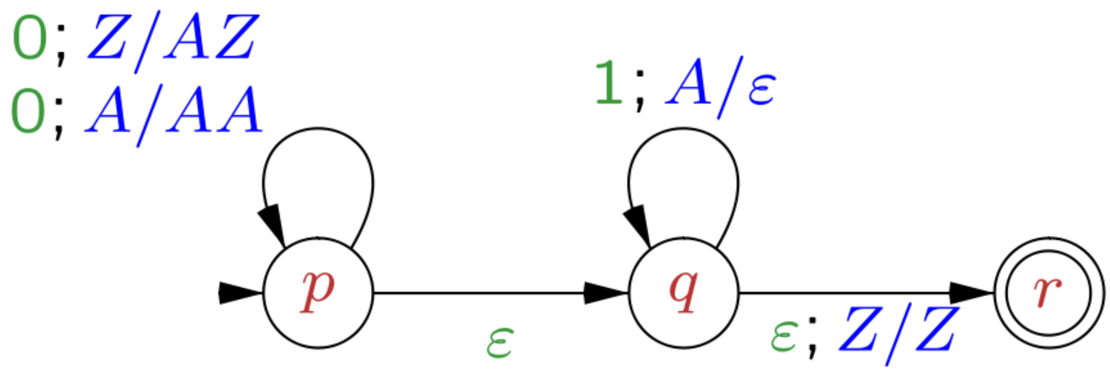
$$
\begin{array}{llll}
(p, & 0011, & Z) & \vdash \\
(p, & 011, & AZ) & \vdash \\
(p, & 11, & AAZ) & \vdash \\
(q, & 11, & AAZ) & \vdash \\
(q, & 1, & AZ) & \vdash \\
(q, & \varepsilon, & Z) & \vdash \\
(r, & \varepsilon, & Z) &
\end{array}
$$



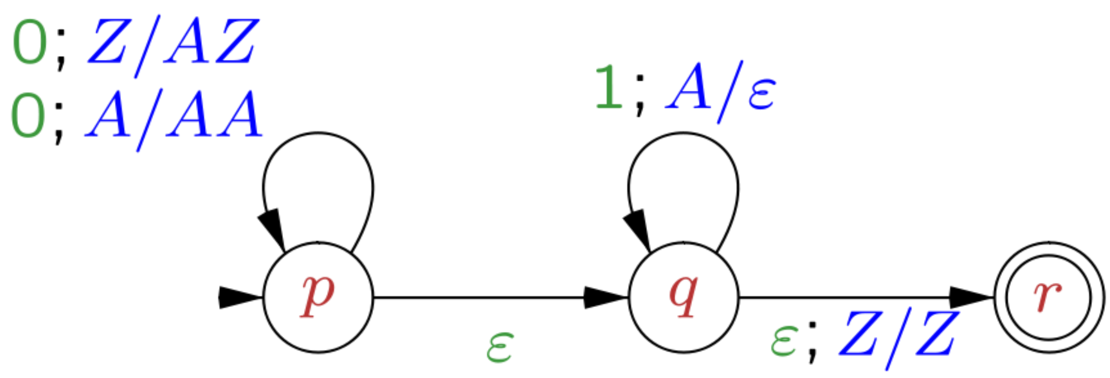
$$
\begin{array}{lrrl}
(p, & 0011, & Z) & \vdash \\
(p, & 011, & AZ) & \vdash \\
(p, & 11, & AAZ) & \vdash \\
(q, & 11, & AAZ) & \vdash \\
(q, & 1, & AZ) & \vdash \\
(q, & \varepsilon, & Z) & \vdash \\
(r, & \varepsilon, & Z) &
\end{array}
$$



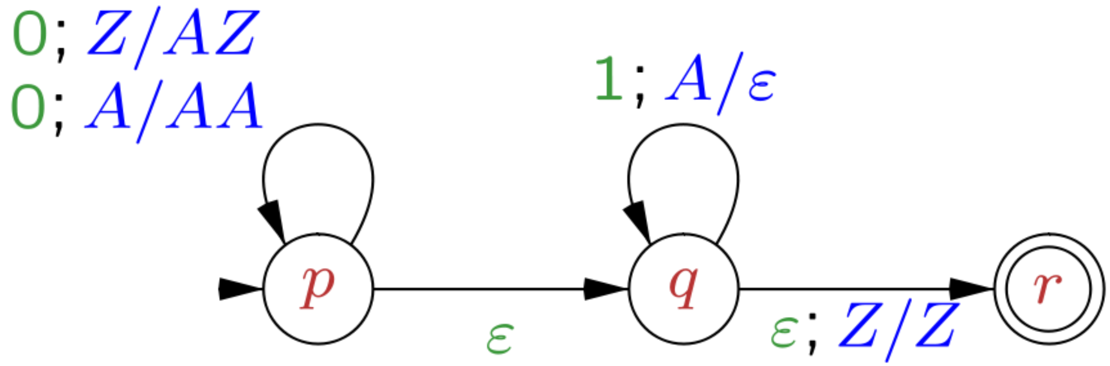
$$(p,\ 001,\qquad Z)\ \vdash$$



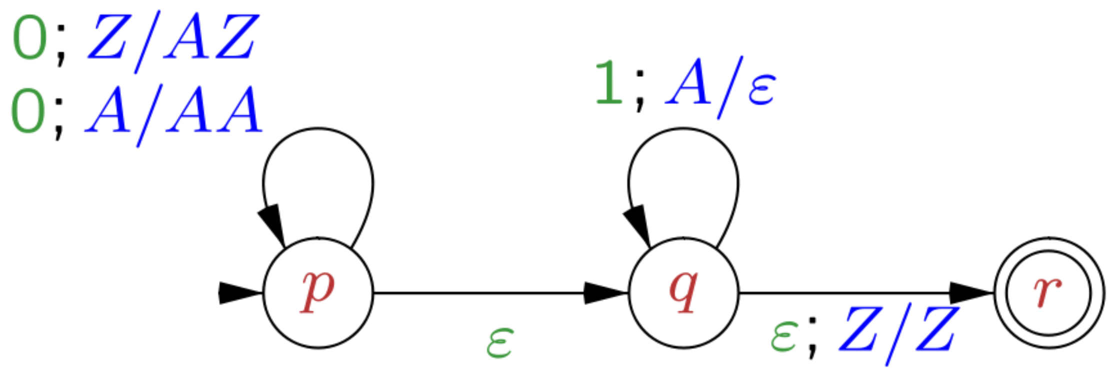
$$(p,\ 001,\ Z) \vdash$$
$$(p,\ 01,\ AZ) \vdash$$



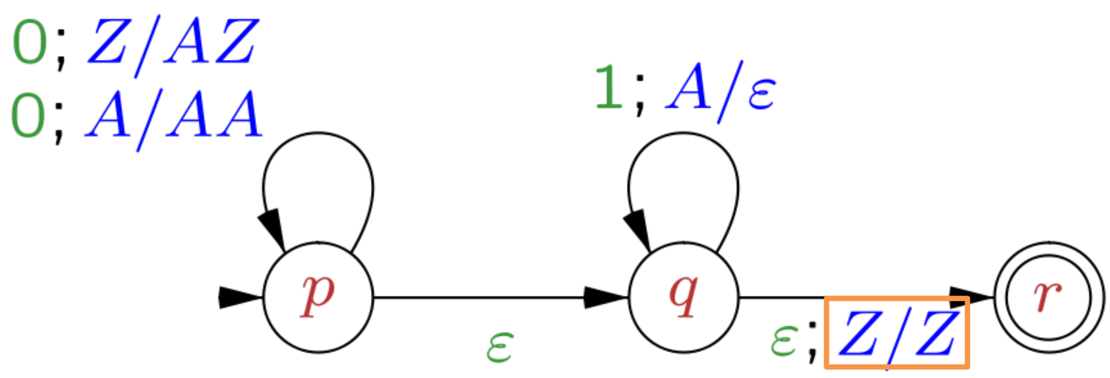
$$\begin{aligned}
&(p,\ 001,\ Z) \vdash \\
&(p,\ 01,\ AZ) \vdash \\
&(p,\ 1,\ AAZ) \vdash
\end{aligned}$$



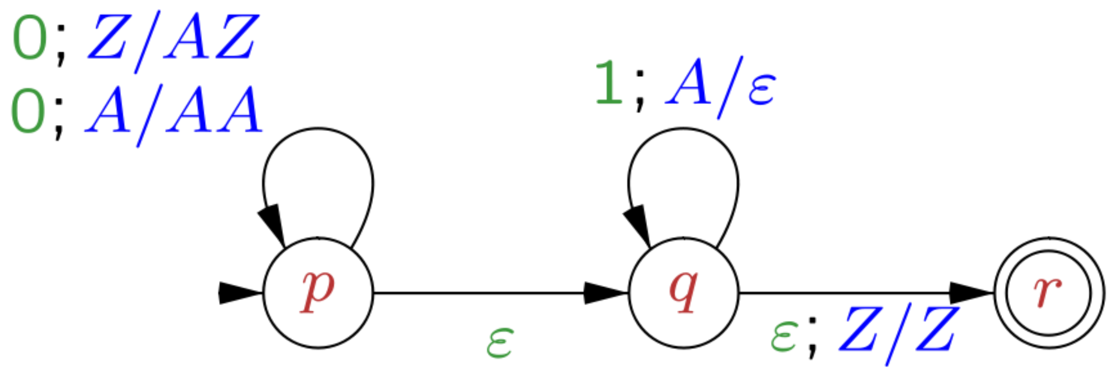
$$
\begin{aligned}
(p,\ 001,\ Z) &\vdash \\
(p,\ 01,\ AZ) &\vdash \\
(p,\ 1,\ AAZ) &\vdash \\
(q,\ 1,\ AAZ) &\vdash
\end{aligned}
$$



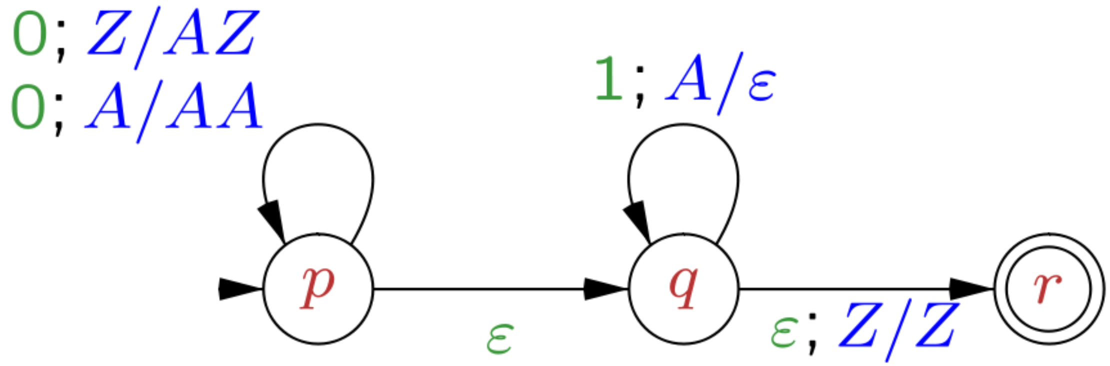
$$
\begin{array}{llrrl}
(p, & 001, & Z) & \vdash \\
(p, & 01, & AZ) & \vdash \\
(p, & 1, & AAZ) & \vdash \\
(q, & 1, & AAZ) & \vdash \\
(q, & \varepsilon, & AZ) & \vdash
\end{array}
$$



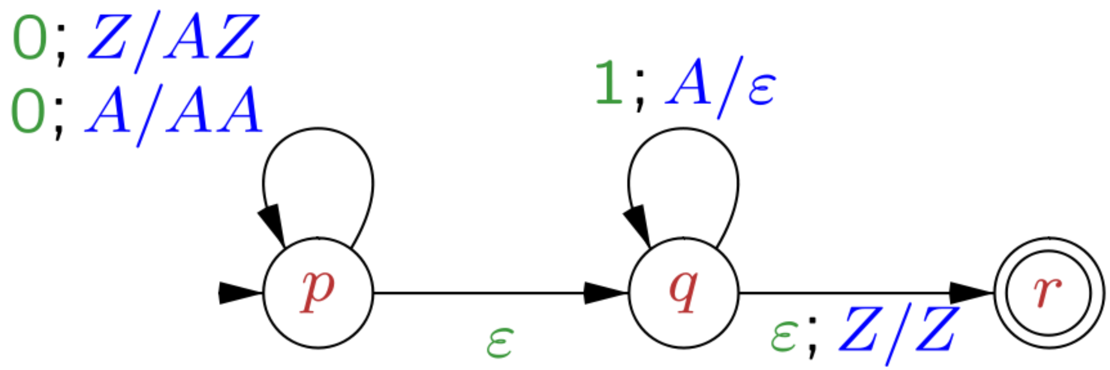
$$
\begin{aligned}
(p,\ 001,\ Z) &\vdash \\
(p,\ 01,\ AZ) &\vdash \\
(p,\ 1,\ AAZ) &\vdash \\
(q,\ 1,\ AAZ) &\vdash \\
(q,\ \varepsilon,\ AZ) &\vdash
\end{aligned}
$$



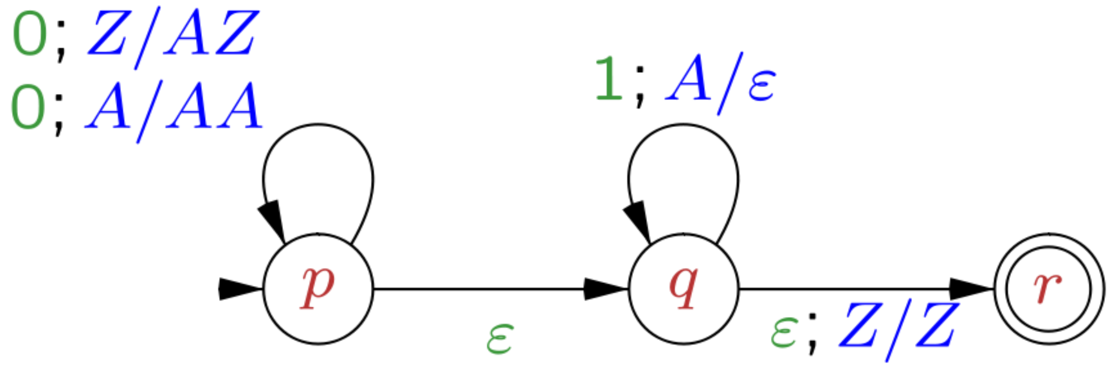
$$(p,\ 00111,\quad Z)\ \vdash$$



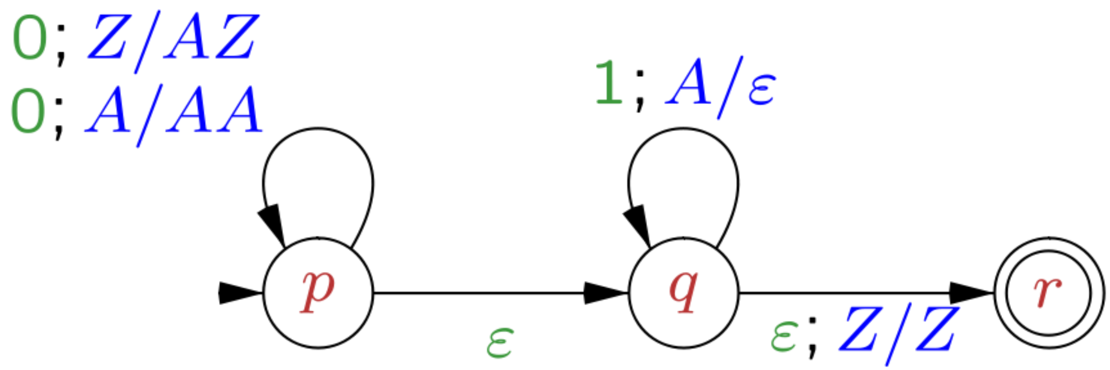
$$
\begin{array}{lrrr}
(p, & 00111, & Z) & \vdash \\
(p, & 0111, & AZ) & \vdash
\end{array}
$$



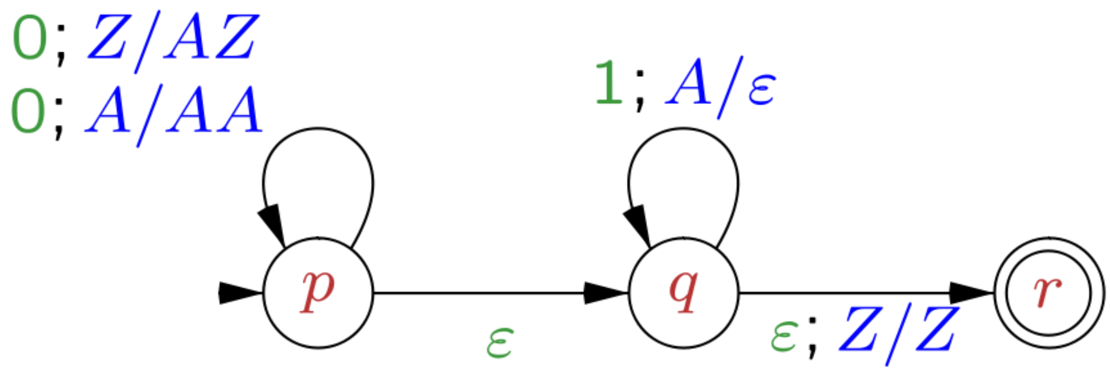
$$
\begin{array}{llrl}
(p, & 00111, & Z) & \vdash \\
(p, & 0111, & AZ) & \vdash \\
(p, & 111, & AAZ) & \vdash
\end{array}
$$



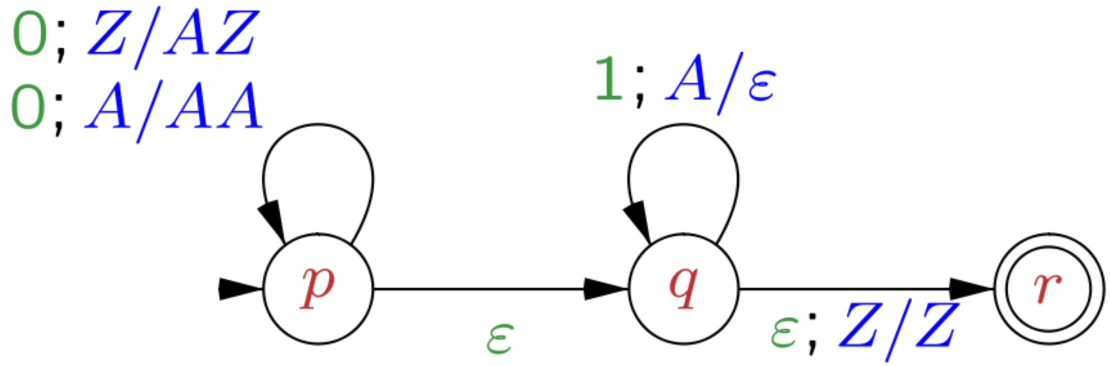
$$
\begin{aligned}
(p,\ 00111,\ Z) &\vdash \\
(p,\ 0111,\ AZ) &\vdash \\
(p,\ 111,\ AAZ) &\vdash \\
(q,\ 111,\ AAZ) &\vdash
\end{aligned}
$$



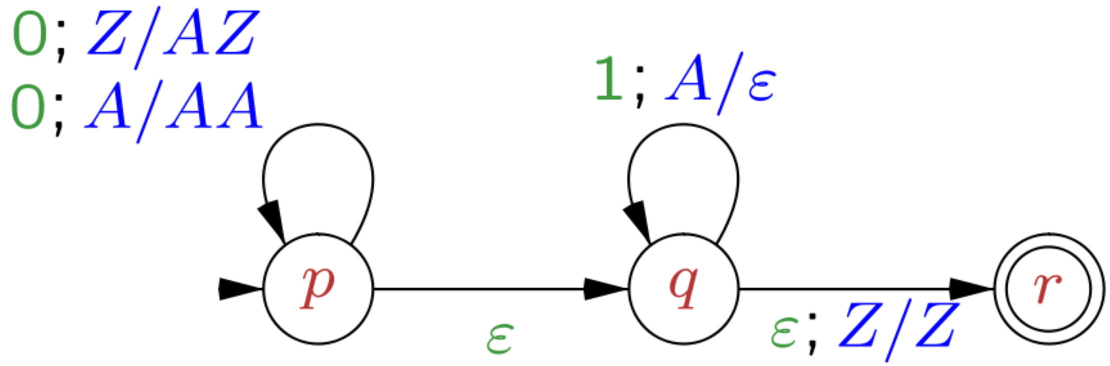
$$
\begin{array}{lrrl}
(p, & 00111, & Z) & \vdash \\
(p, & 0111, & AZ) & \vdash \\
(p, & 111, & AAZ) & \vdash \\
(q, & 111, & AAZ) & \vdash \\
(q, & 11, & AZ) & \vdash
\end{array}
$$



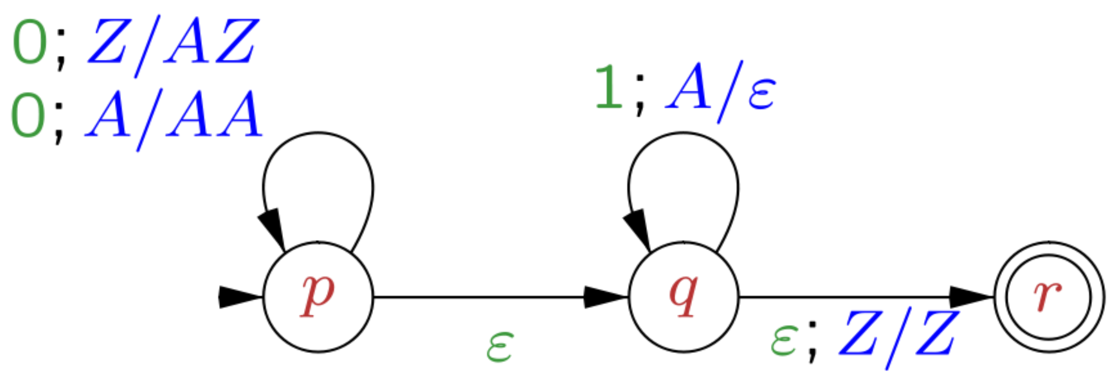
$$
\begin{aligned}
(p,\ 00111,\ Z) &\vdash \\
(p,\ 0111,\ AZ) &\vdash \\
(p,\ 111,\ AAZ) &\vdash \\
(q,\ 111,\ AAZ) &\vdash \\
(q,\ 11,\ AZ) &\vdash \\
(q,\ 1,\ Z) &\vdash
\end{aligned}
$$



$$
\begin{array}{llrrl}
(p, & 00111, & Z) & \vdash \\
(p, & 0111, & AZ) & \vdash \\
(p, & 111, & AAZ) & \vdash \\
(q, & 111, & AAZ) & \vdash \\
(q, & 11, & AZ) & \vdash \\
(q, & 1, & Z) & \vdash
\end{array}
$$



$$
\begin{array}{rrrrl}
(p, & 00111, & Z) & \vdash \\
(p, & 0111, & AZ) & \vdash \\
(p, & 111, & AAZ) & \vdash \\
(q, & 111, & AAZ) & \vdash \\
(q, & 11, & AZ) & \vdash \\
(q, & 1, & Z) & \vdash \\
(r, & 1, & Z) &
\end{array}
$$



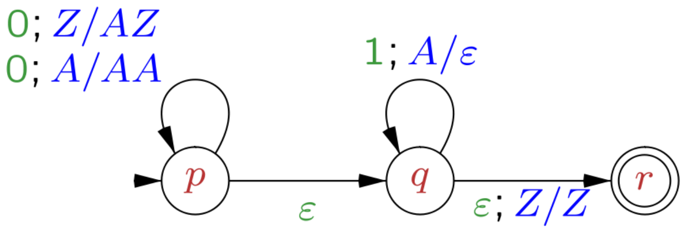
$$
\begin{array}{lrrl}
(p, & 00111, & Z) & \vdash \\
(p, & 0111, & AZ) & \vdash \\
(p, & 111, & AAZ) & \vdash \\
(q, & 111, & AAZ) & \vdash \\
(q, & 11, & AZ) & \vdash \\
(q, & 1, & Z) & \vdash \\
(r, & 1, & Z) &
\end{array}
$$

$$(Q, \Sigma, \Gamma, \delta, q_0, Z, F)$$

$$(Q, \Sigma, \Gamma, \delta, q_0, Z, F)$$

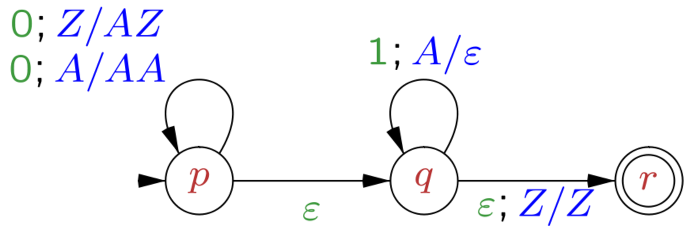
$$(Q, \Sigma, \Gamma, \delta, q_0, Z, F)$$

a finite set of *states*



$(Q, \Sigma, \Gamma, \delta, q_0, Z, F)$

a finite set of *states*



$(Q, \Sigma, \Gamma, \delta, q_0, Z, F)$

a finite set of *states*

{p, q, r}

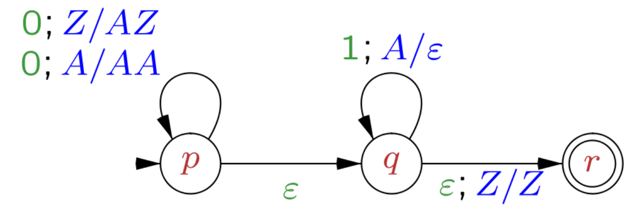
$$(Q, \Sigma, \Gamma, \delta, q_0, Z, F)$$

$$(Q, \Sigma, \Gamma, \delta, q_0, Z, F)$$

a finite set of *input character*

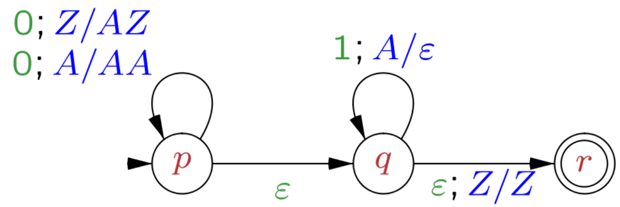
$$(Q, \Sigma, \Gamma, \delta, q_0, Z, F)$$

input alphabet



$(Q, \Sigma, \Gamma, \delta, q_0, Z, F)$

input alphabet



$(Q, \Sigma, \Gamma, \delta, q_0, Z, F)$

input alphabet

{0, 1}

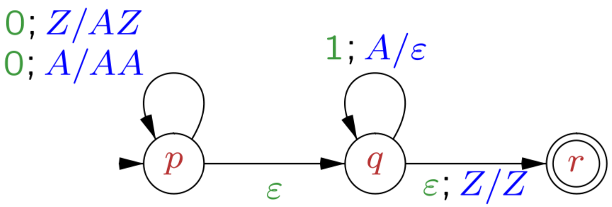
$$(Q, \Sigma, \Gamma, \delta, q_0, Z, F)$$

$(Q, \Sigma, \Gamma, \delta, q_0, Z, F)$

a finite set of *stack character*

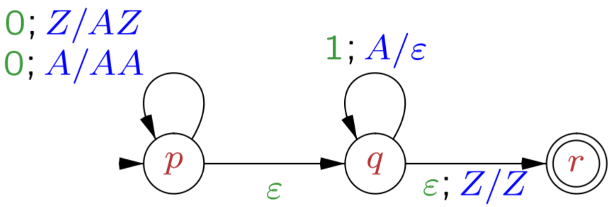
$$(Q, \Sigma, \Gamma, \delta, q_0, Z, F)$$

stack alphabet



$(Q, \Sigma, \Gamma, \delta, q_0, Z, F)$

stack alphabet



$(Q, \Sigma, \Gamma, \delta, q_0, Z, F)$

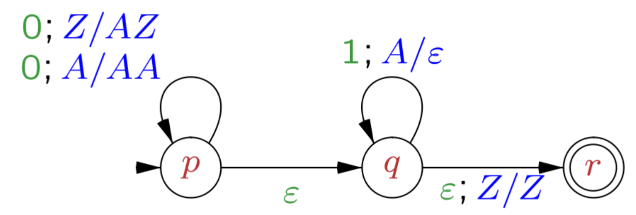
stack alphabet

{A, Z}

$$(Q, \Sigma, \Gamma, \delta, q_0, Z, F)$$

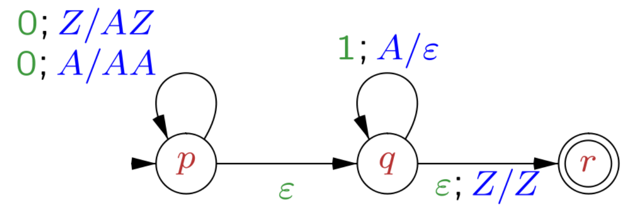
$$(Q, \Sigma, \Gamma, \delta, q_0, Z, F)$$

Transition relation: $Q \times (\Sigma \cup \{\varepsilon\}) \times \Gamma \rightarrow \Gamma^*$



(Q, Σ, Γ, δ, q_0, Z, F)

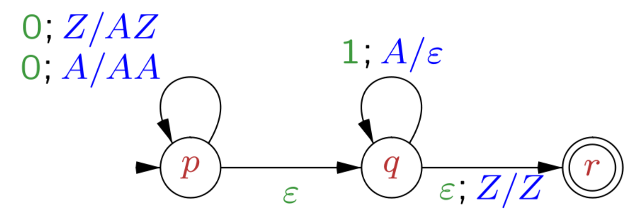
Transition relation: Q x (Σ∪{ε}) x Γ →Γ*



(Q, Σ, Γ, δ, q_0, Z, F)

Transition relation: Q x (Σ∪{ε}) x Γ →Γ*

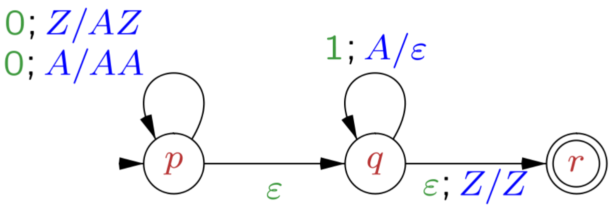
((p,0,Z), (p,AZ))
((p,0,A), (p,AA))



$(Q, \Sigma, \Gamma, \delta, q_0, Z, F)$

Transition relation: $Q \times (\Sigma \cup \{\varepsilon\}) \times \Gamma \to \Gamma^*$

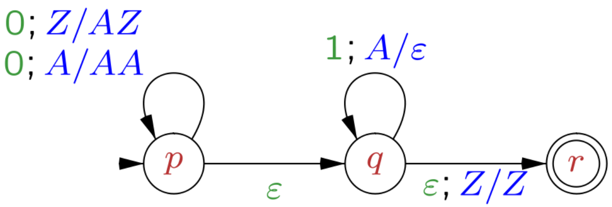
((p,0,Z), (p,AZ))
((p,0,A), (p,AA))
((p,ε,A), (q,A))
((p,ε,Z), (q,Z))



(Q, Σ, Γ, δ, q_0, Z, F)

Transition relation: Q x (Σ∪{ε}) x Γ →Γ*

((p,0,Z), (p,AZ))
((p,0,A), (p,AA))
((p,ε,A), (q,A))
((p,ε,Z), (q,Z))
((q,1,A), (q,ε))



$(Q, \Sigma, \Gamma, \delta, q_0, Z, F)$

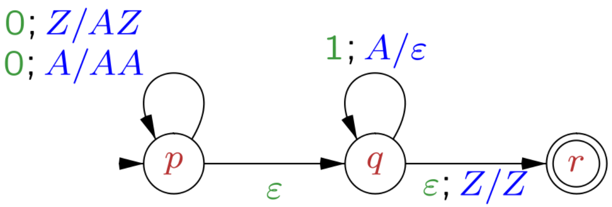
Transition relation: $Q \times (\Sigma \cup \{\varepsilon\}) \times \Gamma \rightarrow \Gamma^*$

((p,0,Z), (p,AZ))
((p,0,A), (p,AA))
((p,ε,A), (q,A))
((p,ε,Z), (q,Z))
((q,1,A), (q,ε))
((q,ε,Z), (r,Z))

$$(Q, \Sigma, \Gamma, \delta, q_0, Z, F)$$

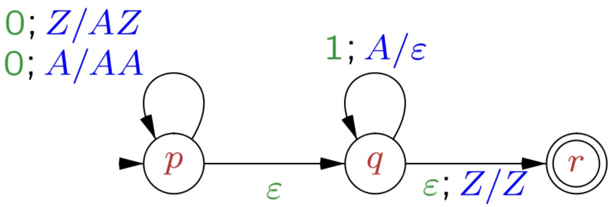
$$(Q, \Sigma, \Gamma, \delta, q_0, Z, F)$$

start state



$(Q, \Sigma, \Gamma, \delta, q_0, Z, F)$

start state



$(Q, \Sigma, \Gamma, \delta, q_0, Z, F)$

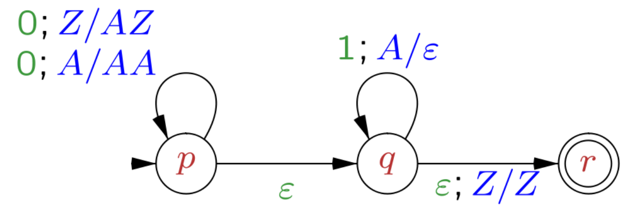
start state

p

$$(Q, \Sigma, \Gamma, \delta, q_0, Z, F)$$

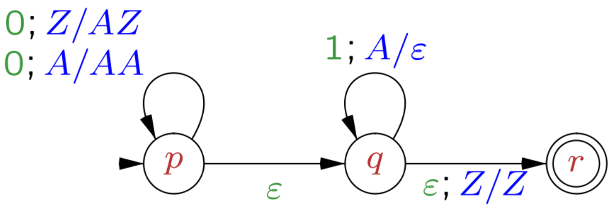
$$(Q, \Sigma, \Gamma, \delta, q_0, Z, F)$$

initial stack symbol



$(Q, \Sigma, \Gamma, \delta, q_0, Z, F)$

initial stack symbol



$$(Q, \Sigma, \Gamma, \delta, q_0, Z, F)$$

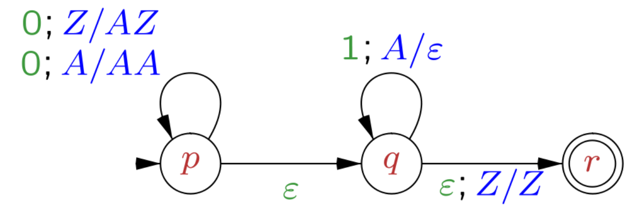
initial stack symbol

Z

$$(Q, \Sigma, \Gamma, \delta, q_0, Z, F)$$

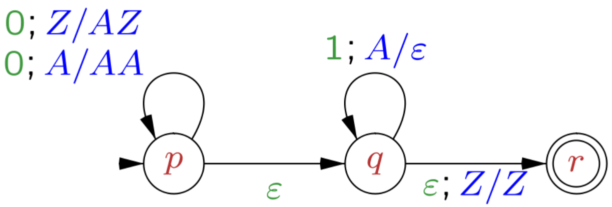
$$(Q, \Sigma, \Gamma, \delta, q_0, Z, F)$$

accepting states



$(Q, \Sigma, \Gamma, \delta, q_0, Z, F)$

accepting states



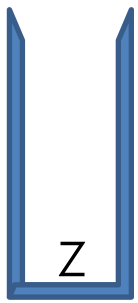
$(Q, \Sigma, \Gamma, \delta, q_0, Z, F)$

accepting states

{r}

0^n1^n

000….0111….1



000....0111....1

000....0111....1

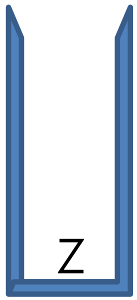
000....0111....1

000....0111....1

000....0111....1

$0^n 1^n 2^n$

000...0111...1222...2



000...0111...1222...2

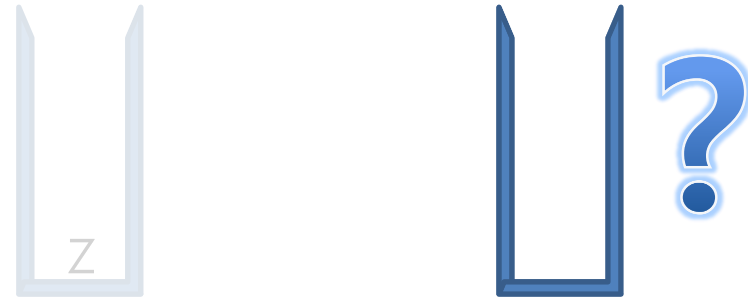
000...0111...1222...2

000...0111...1222...2

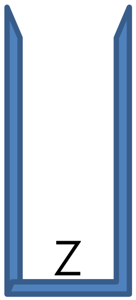
000...0111...1222...2

000…0111…1222…2

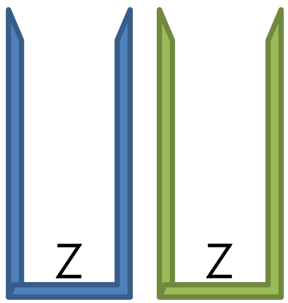
000...0111...1222...2



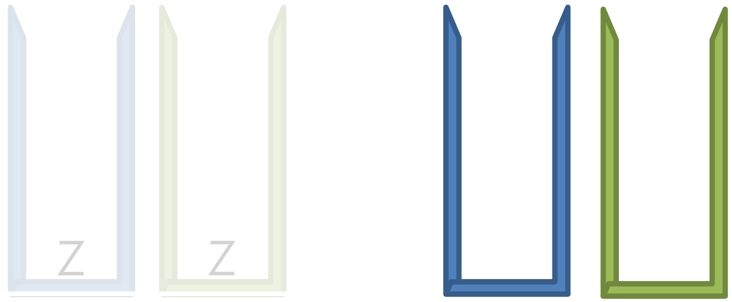
000...0111...1222...2



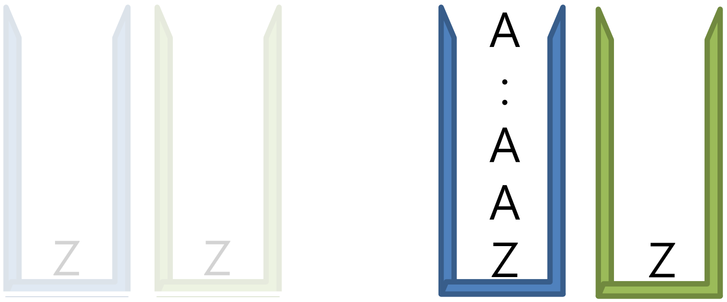
000...0111...1222...2



000…0111…1222…2



000...0111...1222...2



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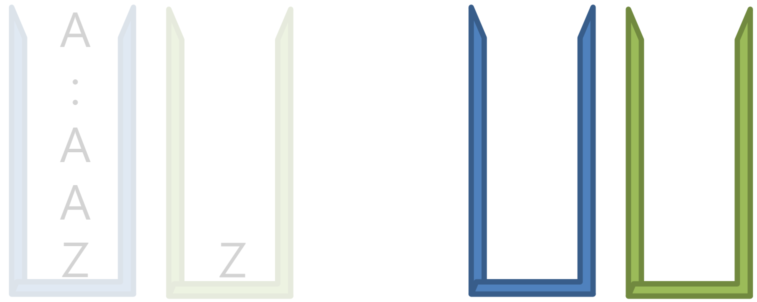

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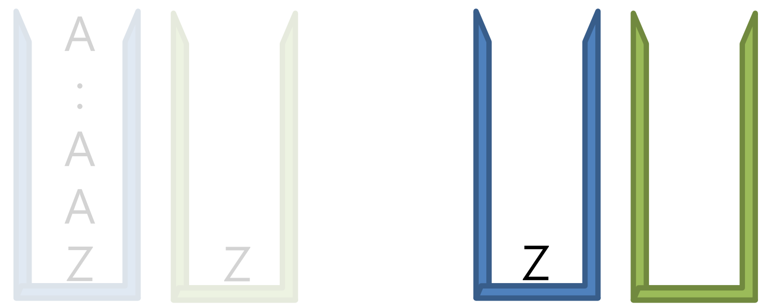

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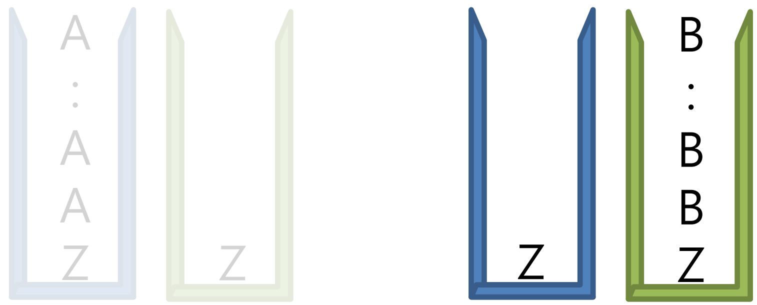

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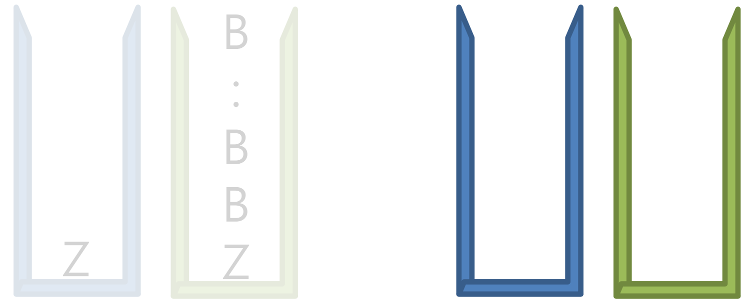

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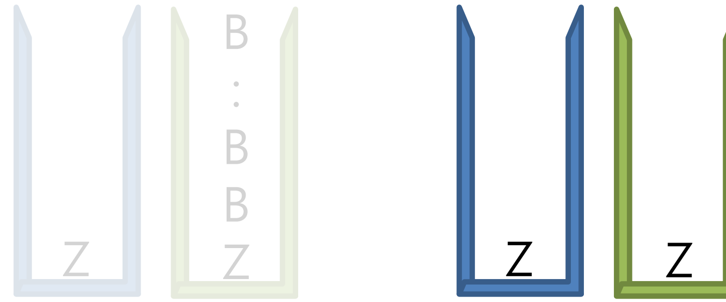

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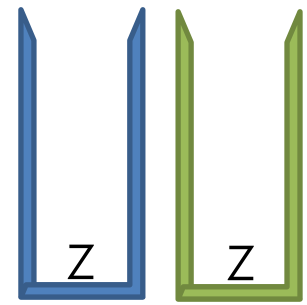

000...0111...1222...2

$0^n 1^n 2^n$

Turing Machine

000111222

bbbbbbbbb000111222bbbbbbbbb

bbbbbbbb$q_0$000111222bbbbbbbb

$q_0000111222$

$0q_1 00111222$

$00q_1 0111222$

$000q_1111222$

$0001q_1 11222$

$00011q_11222$

$000111q_1222$

$000111q_2222$

$00011q_3 3222$

$0001q_3 13222$

$000q_3113222$

$00q_30113222$

$0q_3 00113222$

$q_3000113222$

$q_4000113222$

b$q_1$00113222

$b0q_1 0113222$

b00$q_1$113222

b001$q_1$13222

$b0011q_13222$

$b0011q_2 3222$

$b001q_3 33222$

b00$q_3$133222

b0$q_3$0133222

b$q_3$00133222

b$q_4$00133222

bb$q_1$0133222

bb0$q_1$133222

bb01$q_1$33222

$bb01q_2 33222$

bb0$q_3$333222

$bbq_30333222$

bb$q_4$0333222

bbb$q_1$333222

bbb$q_2$333222

bbb$q_5$333222

bbb333222q_5

bbb333222q_6

bbb33322q_7b

bbb$q_8$33322b

bbbb$q_7$3322b

bbbb3322q_7b

bbbb$q_8$332bb

bbbbb$32q_7$bb

bbbbb$q_8$3bb

bbbbbb$q_7$bb

bbbbbb$q_8$bb